# Selected topics from graph theory 

Final take-home exam

Date: May 15, 2016 Due: May 22, 2016.

Since there is a strict due time for the grades at BSM, I hope you can submit electronically. I recommend pdf compiled from latex. Let me know if you can't do that.
1.1 Problem. Let $G$ be a planar graph in which the outer face is a pentagon, the other faces are triangles, and all nodes in its interior have even degree. Prove that exactly two, adjacent nodes of the pentagonal face have odd degree.
1.2 Problem. Prove that an antipodal map $S^{d} \rightarrow S^{d}$ cannot be homotopic to the constant function mapping everything to the north pole.
1.3 Problem. Let $u_{1}, \ldots, u_{n} \in S^{2}$. Prove that the length of the closed spherical polygon with vertices $u_{1}, u_{2}, \ldots, u_{n},-u_{1}, \ldots,-u_{n}$ is at least $2 \pi$. (The edges of the polygon are the shortest arcs connecting consecutive vertices in this cyclic order.)
1.4 Problem. Determine the Möbius function of the lattice $L$ whose elements are pairs of integers $(x, y)$ with $1 \leq x, y \leq n$, where the partial order is defined by $(x, y) \leq(u, v)$ iff $x \leq u$ and $y \leq v$.
1.5 Problem. Let $L$ be a finite lattice, and suppose that it has an element $x$ that has no complement (i.e., no element $\bar{x}$ such that $x \wedge \bar{x}=0$ and $x \vee \bar{x}=1$ ). Prove that the chain complex $\mathcal{C}(L \backslash\{0,1\})$ is contractible.

