SUMMER SCHOOL IN MATHEMATICS

EÖTVÖS LORÁND UNIVERSITY

BUDAPEST, HUNGARY

June 17−21, 2024

Optimal transport: discrete and continuous

Kristóf Bérczi (ELTE, Budapest):
Network flows and applications

Tamás Király (ELTE, Budapest):
Matching games with transferable and non-transferable utility

Lorenzo Portinale (Hausdorff Center, Bonn):
Optimal transport and aplications to gradient flows

Tamás Titkos (Corvinus University & Rényi Institute, Budapest):
An introduction to classical optimal transport

Dániel Virosztek (Rényi Institute, Budapest):
Selected topics in quantum optimal transport

Budapest, June 2024

Summer School in Mathematics

June 17–21, 2024

Eötvös Loránd University, Budapest, Hungary
in cooperation with
Alfréd Rényi Institute of Mathematics, Budapest, Hungary

OPTIMAL TRANSPORT: DISCRETE AND CONTINUOUS

For graduate and senior undergraduate students

Minicourses:

- **Kristóf Bérczi (ELTE, Budapest):** ***Network flows and applications***
- **Tamás Király (ELTE, Budapest):** ***Matching games with transferable and non-transferable utility***
- **Lorenzo Portinale (Hausdorff Center, Bonn):** ***Optimal transport and applications to gradient flows***
- **Tamás Titkos (Rényi Institute, Budapest):** ***An introduction to classical optimal transport***
- **Dániel Virosztek (Rényi Institute, Budapest):** ***Selected topics in quantum optimal transport***

summerschool@math.elte.hu
https://www.math.elte.hu/summerschool

Registration deadline: June 7, 2024

OPTIMAL TRANSPORT: DISCRETE AND CONTINUOUS
SUMMER SCHOOL IN MATHEMATICS, BUDAPEST, JUNE 17–21, 2024

	Monday, June 17	Tuesday, June 18	Wednesday, June 19	Thursday, June 20	Friday, June 21
9.00 – 10.30	T. TITKOS: *An introduction to classical optimal transport (1)*	T. TITKOS *An introduction to classical optimal transport (2)*	K. BÉRCZI: *Network flows and applications (1)*	K. BÉRCZI: *Network flows and applications (2)*	L. PORTINALE: *Optimal transport and applications to gradient flows (3)*
10.30 – 11.00	C O F F E E / R E F R E S H M E N T				
11.00 – 12.30	T. KIRÁLY: Matching games with transferable and non-transferable utility (1)	T. KIRÁLY: Matching games with transferable and non-transferable utility (2)	L. PORTINALE: *Optimal transport and applications to gradient flows (2)*	T. KIRÁLY: Matching games with transferable and non-transferable utility (3)	D. VIROSZTEK: *Selected topics in quantum optimal transport (3)*
12.30 -- 14.00	L U N C H				
14.00 -- 15.30	L. PORTINALE: *Optimal transport and applications to gradient flows (1)*	D. VIROSZTEK: *Selected topics in quantum optimal transport (1)*	CAVE TOUR (PÁLVÖLGYI CAVE)	D. VIROSZTEK: *Selected topics in quantum optimal transport (2)*	
15.30 – 18.00	WELCOME PARTY / PIZZA	STUDENT WORK PRESENTATIONS		BIKE TOUR	
18.00 – 19.30					

CONTENTS

LIST OF REGISTERED PARTICIPANTS

1. **Bahar Baturoglu** – University of British Columbia, Vancouver (*Canada*)
2. **Gergely Bunth** – Alfréd Rényi Institute of Mathematics, Budapest (*Hungary*)
3. **Mohamed Chakib** – Eötvös Loránd University, Budapest (*Hungary*)
4. **Bentley Eidem** – Université Libre de Bruxelles, Brussels (*Belgium*)
5. **Ábel Göde** – Eötvös Loránd University, Budapest (*Hungary*)
6. **Omar Haddad** – Cité University, Paris (*France*)
7. **Klára Karasová** – Charles University, Prague (*Czech Republic*)
8. **Tamás Kuremszki** – Corvinus University, Budapest (*Hungary*)
9. **David Li** – University Heidelberg, Heidelberg (*Germany*)
10. **Lydia Mirabel Mendoza Cadena** – Eötvös Loránd University, Budapest (*Hungary*)
11. **Balázs Tibor Morvay** – Budapest University of Technology and Economics, Budapest (*Hungary*)
12. **László Palásti** – Corvinus University, Budapest (*Hungary*)
13. **Pijus Petkevičius** – Vilnius University, Vilnius (*Lithuania*)
14. **József Pitrik** – Budapest University of Technology and Economics, Budapest (*Hungary*)
15. **Richárd Simon** – Budapest University of Technology and Economics, Budapest (*Hungary*)
16. **Eric Ströher** – University of Bern, Bern (*Switzerland*)
17. **Luca Szegletes** – Budapest University of Technology and Economics, Budapest (*Hungary*)
18. **Barnabás Szűcs** – Corvinus University, Budapest (*Hungary*)
19. **Zsigmond Tarcsay** – Eötvös Loránd University, Budapest (*Hungary*)
20. **Szabolcs Torma** – Budapest University of Technology and Economics, Budapest (*Hungary*)
21. **Abris Wunderlich** – Corvinus University, Budapest (*Hungary*)

PREFACE

The first international summer school in mathematics, organized by the Institute of Mathematics at Eötvös Loránd Universty in Budapest, Hungary, took place in 2013. Since then a series of similar one week events was organized each year (with the exception of the two COVID-years, i.e. 2020 and 2021). Starting from the second year the schools were concentrating on one particlular topic (general discrete mathematics, algorithms, graph limits, algebraic geometry and topology, number theory, real analysis etc.) A large portion of related materials of these schools can be found at the archives of the website of the series:

`http://www.math.elte.hu/summerschool/?page=download`

The summer school organized in 2024 was the 10th in this series. It took place at the Lágymányos Campus of Eötvös Loránd University in Budapest between June 17 and 21, 2024. The title of the school was *Optimal transport: discrete and continuous.*

The lecturers were Kristóf Bérczi (Eötvös Loránd University, Budapest), Tamás Király (Eötvös Loránd University, Budapest), Lorenzo Portinale (Hausdorff Center, Bonn), Tamás Titkos (Corvinus University and Rényi Institute, Budapest) and Dániel Virosztek (Rényi Institute, Budapest).

Handwritten course notes were made available to the participants for some of the lectures, either before or after the lectures. The present booklet besides containing the preliminary abstracts of these talks, contains the schedule of lectures plus the list of participants. This volume can be downloaded from the same website as the notes of the previous schools.

We wish to thank Eötvös Loránd University and the Alfréd Rényi Mathematical Institute for financial support. We would also like to express our gratitude to all lecturers but also to the audience whose active participation makes the whole series of summerschools meaningful.

Budapest, September 5, 2024

István Ágoston
organizer
Email: `summerschool@math.elte.hu`

1. KRISTÓF BÉRCZI (ELTE, Budapest)
Network flows and applications

Network flow theory provides a basic tool to treat conveniently various graph characterization and optimization problems, including the degree-constrained subgraph problem in a bipartite graph. Another general framework in graph optimization is matroid theory. For example, the problem of extending k given subtrees of a graph to k disjoint spanning trees can be solved with the help of matroids. A common generalization of these two big branches of combinatorial optimization is the theory of submodular flows, initiated by Edmonds and Giles. This covers not only the basic results on maximum flows and min-cost circulations from network flow theory and weighted matroid intersection from matroid theory, but also helps solving significantly more complex graph optimization problems. In this minicourse, we give an overview of the most fundamental results and techniques of this area.

2. TAMÁS KIRÁLY (ELTE, Budapest)
Matching games with transferable and non-transferable utility

In the discrete version of the optimal transport problem, optimality can be certified by a simple witness that has a natural interpretation in cooperative game theory as profit sharing between partners. Cooperative games involving the distribution of profit (or utility) among partners are called transferable utility games, and we will discuss some notable examples including matching games as well as games based on other combinatorial optimization problems. The lecture series will also offer an introduction to matching games with non-transferable utility. These include the famous stable marriage problem of Gale and Shapley and its generalizations, as well as more recent solution concepts like nearly stable matchings and popular matchings.

3. LORENZO PORTINALE (Hausdorff Center, Bonn)
Optimal transport and aplications to gradient flows

In this lecture we discuss some of the intriguing connections between the theory of optimal transport and a special class of partial differential equations. In their seminal work, Jordan, Kinderlehrer, and Otto understood that the heat flow equation in Euclidean space can be suitably interpreted as a infinite dimensional gradient-flow equation of the entropy functional with respect to the quadratic Wasserstein distance. One of the challenges of this approach is to provide a rigorous way to handle gradient flows in a general framework such as the one of metric spaces. There are different notions of solutions of a gradient-flow equation in this setting, and starting from the example of the heat flow we will discuss the motivations and the relations between them. Finally, we are going to discuss the advantages that such a variational formulation of certain PDEs brings, including the applications to discrete-to-continuum approximation problems and the generalisation of these ideas to possibly nonlinear and less regular settings.

4. TAMÁS TITKOS (Corvinus University & Rényi Institute, Budapest)
An introduction to classical optimal transport

In this lecture series, I will give a gentle introduction to the classical optimal transport problem. The main topics will be:

- Monges problem, basic examples demonstrating that the transport problem is ill-posed
- Kantorovichs relaxation, the existence of optimal transport plans
- Kantorovich duality
- Breniers theorem on the existence of optimal transport maps transport-related metrics, basic properties of Wasserstein spaces applications in pure mathematics and in applied sciences.

I do not assume any prior knowledge of the theory. In fact, I will start from scratch, recalling some basic notions of measure and integration.

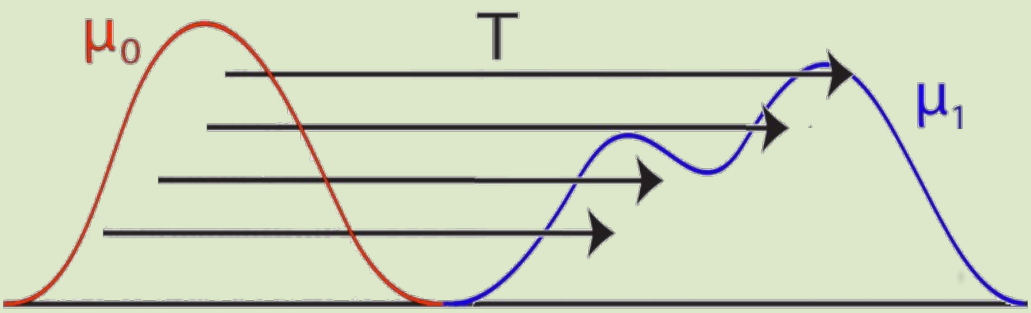
5. DÁNIEL VIROSZTEK (Rényi Institute, Budapest)
Selected topics in quantum optimal transport

Although the theory of classical optimal transport has been playing an important role in mathematical physics (especially in fluid dynamics) and probability since the late 80s, concepts of optimal transportation in quantum mechanics have emerged only very recently. It is a general phenomenon that concepts and notions well-established in the classical commutative world do not have a unique best extension in the non-commutative world, but there are many possible ways of generalization with pros and cons. This is the case concerning optimal transportation as well: non-commutative optimal transport is a flourishing research field these days with several different approaches.

During this lecture series, we will restrict our attention to two concepts of mass transportation in quantum mechanics: one relying on quantum channels (pioneered by De Palma and Trevisan) and one relying on quantum couplings (pioneered by Caglioti, Golse, Mouhot, and Paul). We will discuss fundamental results like the quantum Kantorovich duality and surprising phenomena like quantum optimal transport is cheaper. Then, we turn to the metric side of the theory. A common feature of the quantum optimal transport concepts above is that the induced quantum Wasserstein distances are not genuine metrics. In particular, a state may have a positive distance from itself. We will highlight the physical reasons for this phenomenon from Heisenberg's uncertainty relation to the connection of the self-distance with the WignerYanase skew information. Finally, we will review the positive results regarding the metric properties of (natural variants of) the aforementioned quantum Wasserstein distances.

Kristóf Bérczi **Tamás Király** **Lorenzo Portinale**

Tamás Titkos **Dániel Virosztek**



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