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CONTAGION OF CREDIT RATING DOWNGRADES IN FINANCIAL NETWORKS

Master's Thesis

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1. INTRODUCTION

Financial contagion has been a subject of great importance in recent years, especially since the increased attention given to systemic risk and the joint behaviour of the financial system. This has caused a growing interest in the field of network theory and how its concepts and techniques may be applied to model financial networks that incorporate the connections and interdependencies among a set of organizations such as banks, firms or countries. The aim of this thesis is to investigate the existence and behaviour of network-based effects in the context of credit rating migration. A basic question the thesis wants to address is whether a credit rating downgrade of a given organization can have a detrimental effect on the rating of others simply because they are connected along some form of economic or financial dependencies in a network. More generally, the question of interest is how contagion can lead to a cascade of downgrades and what attributes of the network affect the likelihood and magnitude of such a cascade. Understanding these effects is crucial, as in today's globalized world one cannot focus on the credit risk of organizations in isolation, but has to consider all direct and indirect effects in a networked setting.

The model used in this thesis to study contagion of a credit rating downgrade in a network of interdependent organizations is an extension of the methodology presented in Elliott et al (2014). They model the dependencies as cross-holdings: organizations own shares in each other, thus a decrease in the value of one organization can have a negative impact on the value of other members of the network. They investigate how a cascade of failures can emerge as a result of a shock in such a network and find that a first failure can occur when the level of cross-holdings is moderate (medium integration), while this can lead to a large number of further failures if the network is connected enough for a cascade to propagate, but not so connected that the negative effect of a failure is spread out among many organizations in the network (medium diversification).

The key contribution of this thesis is to move beyond the binary default/survival approach of Elliott et al and to allow organizations to have credit ratings, for example, ranging from default to AAA. This novelty allows for two important aspects of the extended model. First, in the equilibrium and organization does not need to suffer a default in order to be negatively impacted by an adverse shock in its neighbourhood. Instead, it can suffer one or multiple non-default downgrades which is what is observed much more often in real financial systems where defaults are rare. Second, in the expanded model the impact of initial distribution of ratings can be studied, potentially shading light on the importance (or lack thereof) of being integrated in a neighbourhood of high-quality borrowers.

The effect of an initial downgrade can be traced on ratings throughout the network. A first downgrade can be the result of a negative shock to some organization's value whose rating drops as a consequence. The organization suffers a downgrade cost which is an additional decrease in its value. A real world example of this is an increase in the cost of capital upon downgrade. This cost impacts other organizations through the network of cross-holdings and can potentially trigger a cascade of downgrades. The model is used to assess the effect of different properties of the network. Besides measures of integration and diversification, the average rating of organizations is one of the main focus the thesis. Here the question of interest is how the extent of a cascade differ for a network with a large number of highly rated organizations compared to one with more organizations of a bad rating. The inclusion of multiple ratings also allow for the decomposition of a cascade to different types of downgrades based on the original and the resulting rating. The effects of the parameters of interest are split up according to these types to properly understand the mechanism of contagion in the model.

Simulations of the model using random networks confirm that findings of Elliott et al (2014) extend to the world where multiple credit rating are possible, but also provide new insights. In particular, the results show that there is a region of high diversification, where the network is diversified enough so that a cascade of failures does not occur, but a cascade of single-rating downgrades that affect some organizations of a high rating can potentially emerge. This is an additional, second sweet spot for contagion of credit rating downgrades. The mechanism behind this result is that increasing diversification helps to avoid defaults first, but normal downgrades that do not result in failure only disappear when diversification increases further, as they are modelled to be more likely than defaults to be consistent with attributes of real-world downgrades.

The second main finding of the thesis is that when the ingredients are present for a cascade of downgrades, a network mostly consisting of organizations with good ratings is more susceptible to contagion than a network of mostly bad organizations. In the first sweet spot of medium diversification, a more extensive cascade of defaults emerges if the network has more good organizations, while in the second sweet spot of high diversification, a greater percentage of the good organizations suffer a single-rating downgrade compared to a network where the majority of organizations is rated junk. The intuition is that if a cascade can emerge and contagion can propagate through the network, then a network of organizations with higher ratings can lose more ratings overall and thus the cumulative downgrade cost to the economy can be higher, which in turn leads to more downgrades in general.

Apart from simulation techniques, the thesis overcomes the problem of the absence of detailed, bilateral data on financial dependencies between organizations by utilizing international export data from the The United Nations Commodity Trade Statistics Database to construct the trade matrix as an estimate for the underlying true network of cross-holdings of countries of the world. Then, an illustration is given on how to model can be applied in this real network and how it can be used to make predictions about change in the S&P credit ratings of sovereigns that may arise due to a macroeconomic shock.

This thesis fits into the vast literature on financial contagion. Besides the work of Elliott et al (2014), there is a large number of papers that study the behaviour of contagion and cascades of failures in financial networks using various methodologies. Acemoglu, Ozdaglar and Tahbaz-Saehi (2015) model a network of interbank lending. They focus on different sizes of shocks that can lead to a cascade of defaults and investigate the types of network structures that are most resilient and most susceptible to contagion. They find that for limited shocks, a complete network is the least fragile, while for large shocks, less connections provide a form of protection from failures. Gouriéroux, Héam, and Monfort (2012) model the balance sheet of a set of institutions where the financial interdependencies are given by cross-holdings of shares and debt. They define how cascades of defaults may arise as a result of different exogenous shocks and quantify their extent by comparing the number of failures in a specific real network compared to a network without any connections. They also show how the effect of the shock can be split up to a direct part and a network-based contagion component. Gai and Kapadia (2010) consider a random network of interbank claims that is characterized by its degree-distribution. They also study a cascade of failures and contagion effects of an exogenous shock or default and find that although their development maybe unlikely, their effects can be excessive if they emerge. They also observe that these effects can potentially depend on where the shock hits in the network. Glasserman and Young (2016) provide a survey on this issue of the various mechanics and ingredients of financial contagion of defaults and on the possibilities of how its extent can be measured.

The main criticism of such research is that financial contagion is studied with a focus on cascades of defaults which rarely occur in reality. Policymakers usually intervene at an early stage and prevent the formation of a cascade in the first place. On the other hand, credit rating changes are more common in financial networks, thus give a more plausible framework to study the nature of financial contagion and allow for more realistic conclusions. Because of the greater interest in negative contagion effects, one tends to focus on cascades of downgrades first naturally. However, it may also be interesting whether the effects are different for upward credit rating migration. The study of Grinis (2015) is a rare example of a paper that focuses on financial contagion in the context of credit rating changes. A further contribution is to also consider upgrades. Here, the analysis of financial contagion is based on the epidemiology literature and credit ratings are modelled as infection stages of a disease, where in each stage an institution gets downgraded one rating with a certain probability. This methodology allows one to model how an exogenous rating change of given organization in a network may affect that creditworthiness of others. Application of the theoretical model to a network of the Eurozone countries based on data on cross-border Total Portfolio Investment flows shows how the most important countries and how those most vulnerable to downgrades can be identified. As for positive contagion effects of upgrades, the author finds that in theory it could offset negative effects, but in this particular real network it cannot outweigh them. Although the methodology is quite different from that of this thesis, the idea to focus on network-based effects of credit rating changes is the same. Nevertheless, the results are hard to compare, as the main focus of this thesis is on simulation of random networks and application to real networks only serve as an illustration.

The rest of this thesis is organized as follows. Section 2 describes the model. Section 3 presents the framework for simulations and the main results of the thesis. Section 4 illustrates how the model can applied to real networks. Section 5 concludes.

2. THE MODEL AND HOW A CASCADE OF DOWNGRADES CAN EMERGE

As mentioned before, the model used in this thesis is an extension of the one presented in Elliott et al (2014). This section starts with a description of the original model. Then, the changes are presented, which make the model suitable to study the effects of network contagion in the context of credit rating migration. At the end of the section a simple example is given to illustrate the model and how an initial exogenous shock to the economy can lead to a cascade of downgrades.

2.1 The original model

This subsection gives a summary of the methodology presented in Elliott et al (2014). The authors model a network of interdependent organizations (e.g. firms, financial institutions or sovereigns) with cross-holdings between them. That is, besides each organization holding, so-called primitive assets (e.g. some fundamental investments generating streams of net cash flows), they also hold shares of other organizations. Besides shares, the cross-holdings can also be considered in the form any kind of contracts of debt or other liabilities. This way, the value of an organization is not only determined by the values of the primitive assets it holds, but it also depends on the values of other members of the network through the direct and indirect connections incorporated in the network structure. Since the presence of cross-holdings lead to the familiar issue of inflated book values, Elliott et al (2014) derive a formula for the noninflated market value of each organization that also captures how these ultimately depend on the values of all of the primitive assets. In addition, every organization is considered to have a failure threshold, so that if its market value falls below a certain level, the organization defaults and incurs a failure cost, which is an additional, discontinuous drop in its market value. Examples of this are bankruptcy costs, like legal fees, or the cost of fire sale of illiquid assets. At first, such failure can be the consequence of an exogenous shock to some primitive assets. But due to the interconnected system of cross-holdings and the presence of failure costs, an initial default of an organization decreases the values of others and can lead to further failures which again has a negative effect potentially causing more failures, and so on. Hence, it is possible to model cascades of failures and investigate how their likeliness and magnitude is affected by different attributes of the network structure.

Formally, the original model is as follows. The network consists of n organizations. There are m primitive assets, p_k is the market price of asset k. The share of the value of asset k owned by organization i is $D_{ik} \geq 0$ and $\boldsymbol{D}$ is the n x m matrix for which $[\boldsymbol{D}]_{ik} = D_{ik}$. For simplicity, it is often assumed that $n = m$ and $\boldsymbol{D} = \boldsymbol{I}$, so that each organization holds a single, proprietary asset. The cross-holdings are captured by the n x n matrix $\boldsymbol{C}$, whose (i, j)th entry, C_{ij}, is the fraction of organization j held by organization i. $C_{ij} \geq 0$ and for each i $C_{ii} = 0$. The remaining share not owned by other organizations is given by $\hat{C}_{ii} := 1 - \sum_j C_{ji}$. This is assumed to be positive and is held by shareholders of organization i who are outside of the network of cross-holdings. $\widehat{\boldsymbol{C}}$ is the n x n diagonal matrix defined as $\left[\widehat{\boldsymbol{C}}\right]_{ii} = \hat{C}_{ii}$. In this framework of cross-holdings, the value of organization i, denoted by V_i, is the sum of the values of the primitive assets it directly holds, plus the sum of the values of its shares in other organizations, that is

$$V_i = \sum_{k=1}^{m} D_{ik} p_k + \sum_{j=1}^{n} C_{ij} V_j$$

or equivalently in matrix notation

$$\boldsymbol{V} = \boldsymbol{Dp} + \boldsymbol{CV}$$

, where $\boldsymbol{V}$ is the n x 1 vector of the values of the organizations and $\boldsymbol{p}$ is the m x 1 vector of the values of the primitive assets. The solution of this equation is

$$\boldsymbol{V} = (\boldsymbol{I} - \boldsymbol{C})^{-1} \boldsymbol{Dp}$$

, where the inverse exists and is nonnegative. However, Elliott et al (2014) argue that V_i is the inflated book value of organization i and that its noninflated market value, v_i, can be calculated as $\hat{C}_{ii} V_i$, which is the correct measure of value that accumulates to outside shareholders and the relevant measure used throughout the rest of this thesis. Therefore, the appropriate valuation formula is

$$\boldsymbol{v} = \widehat{\boldsymbol{C}}\boldsymbol{V} = \widehat{\boldsymbol{C}}(\boldsymbol{I} - \boldsymbol{C})^{-1} \boldsymbol{Dp} = \boldsymbol{ADp}$$

, where now $\boldsymbol{v}$ is the n x 1 vector of the market values and $\boldsymbol{A} = \widehat{\boldsymbol{C}}(\boldsymbol{I} - \boldsymbol{C})^{-1}$ is the so-called dependency matrix. Its (i, j)th entry, A_{ij}, represents how the value of organization i depends on the values of j's directly held primitive assets. In the model, each organization has a failure threshold $\underline{v}_i$, so that if v_i falls below that, then the organization fails and suffers a discontinuous decrease in its book value , denoted by β_i. With this failure cost incorporated in the model, the previous equation for V_i becomes:

$$V_i = \sum_{k=1}^{m} D_{ik} p_k + \sum_{j=1}^{n} C_{ij} V_j - \beta_i \mathbf{1}_{v_i < \underline{v}_i}$$

, where the indicator $\mathbf{1}_{v_i<\underline{v}_i}$ equals 1 if $v_i < \underline{v}_i$ and 0 otherwise. Let $\boldsymbol{b}(\boldsymbol{v})$ be the n x 1 vector of the failure costs, whose ith element is $b_i(\boldsymbol{v})= \beta_i \mathbf{1}_{v_i<\underline{v}_i}$. Then the solution to the matrix equation of the book values changes to

$$\boldsymbol{V} = (\boldsymbol{I} - \boldsymbol{C})^{-1}(\boldsymbol{Dp} - \boldsymbol{b}(\boldsymbol{v})).$$

Likewise, the new equation for the market values is given as

$$\boldsymbol{v} = \widehat{\boldsymbol{C}}\boldsymbol{V} = \widehat{\boldsymbol{C}}(\boldsymbol{I} - \boldsymbol{C})^{-1}(\boldsymbol{Dp} - \boldsymbol{b}(\boldsymbol{v})) = \boldsymbol{A}(\boldsymbol{Dp} - \boldsymbol{b}(\boldsymbol{v})).$$

The authors argue that there always exists a vector of equilibrium values $\boldsymbol{v}$ that satisfy the last equation, but it is not necessarily unique. If the value of organization i depends on j's and vice versa, then it is possible that in one equilibrium neither of them fail, but in another both organizations fail. The so-called best-case equilibrium and the worst-case equilibrium are the ones with the minimum and the maximum amount of failures, respectively. When looking at how many failures ultimately happen when some organization defaults due to an initial negative shock to its value, one can focus on either equilibrium and derive analogous results in either case (Elliott et al, 2013).

2.2 The modified model

A natural extension of the model is to not limit the state of the organizations to failed or not failed, but to allow for several states of financial health. It is straightforward to include credit ratings in the model and thus study rating migration in a networked setting. For this purpose of this thesis, the model is modified in the following particular way. Other parametrization and generalizations are possible and could be a focus of further research. Suppose that there are an s number of credit ratings an organization can have ranging from the best rating of s (e.g. AAA) to the worst rating of 1 (e.g. default). Consequently, organization i of rating $q > 1$ has $q - 1$ thresholds denoted as $\underline{v}_i^1 < \cdots < \underline{v}_i^{q-1}$. This implies that only downgrades are considered, upward rating changes are excluded from the model. Analogous to the failure cost in the original model, for every threshold there is a corresponding downgrade cost. That is, if v_i falls below $\underline{v}_i^l$ then i is downgraded to a rating of l, and its value decreases by β_i^l. One can think of such downgrade cost as a discontinuous jump in the cost of capital for an organization upon downgrade. A downgrade of multiple ratings is considered as an instantaneous succession of multiple downgrades, so that the costs add up. The modified equation for the book value of i with a rating of $q > 1$:

$$V_i = \sum_{k=1}^{m} D_{ik} p_k + \sum_{j=1}^{n} C_{ij} V_j - \sum_{l=1}^{q-1} \beta_i^l \mathbf{1}_{v_i < \underline{v}_i^l}$$

The solution of the corresponding matrix equation is

$$\boldsymbol{V} = (\boldsymbol{I} - \boldsymbol{C})^{-1} \left(\boldsymbol{Dp} - \sum_{l=1}^{q-1} \boldsymbol{b}^l(\boldsymbol{v}) \right)$$

, where $b_i^l(\boldsymbol{v})$= $\beta_i^l \mathbf{1}_{v_i < \underline{v}_i^l}$. Finally, the modified equation for the market values:

$$\boldsymbol{v} = \widehat{\boldsymbol{C}}\boldsymbol{V} = \widehat{\boldsymbol{C}}(\boldsymbol{I} - \boldsymbol{C})^{-1} \left(\boldsymbol{Dp} - \sum_{l=1}^{q-1} \boldsymbol{b}^l(\boldsymbol{v}) \right) = \boldsymbol{A} \left(\boldsymbol{Dp} - \sum_{l=1}^{q-1} \boldsymbol{b}^l(\boldsymbol{v}) \right)$$

Similarly to the original model, multiple equilibria can arise in the modified model with different ratings of the organizations in different equilibria. Nevertheless, one can again focus on the best-case equilibrium, which is the one with the highest values and highest ratings. The results of this thesis are derived for this equilibrium. In this new framework, it is now possible to trace out the impact of an initial shock that causes the downgrade of some organization and affects the values and ratings of others in the network. An additional innovation of the modified model is that it allows for different initial rating scenarios. That is, the effects might be different depending on the initial distribution of the ratings in the network before the shock hits. This gives a new aspect to study the network-based effects of a shock to an organization's value and creditworthiness.

2.3 A simple example and an algorithm for identifying a cascade of downgrades

Before turning to the main results of the thesis it is useful to illustrate the model through a simple example. First, one has to describe the network structure. Suppose that the network consist of $n = 2$ organizations, with each holding a 60% share of the other. This means that the cross-holdings matrix, $\boldsymbol{C}$, the matrix $\widehat{\boldsymbol{C}}$, and the dependency matrix, $\boldsymbol{A}$**,** are as follows.

$$\boldsymbol{C} = \begin{pmatrix} 0 & 0.6 \\ 0.6 & 0 \end{pmatrix} \qquad \widehat{\boldsymbol{C}} = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix} \qquad \boldsymbol{A} = \widehat{\boldsymbol{C}}(\boldsymbol{I} - \boldsymbol{C})^{-1} = \begin{pmatrix} \frac{5}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{5}{8} \end{pmatrix}$$

Figure 1 represents the network of cross-holdings as a weighted directed graph. It shows how a unit of income generated at one of the organizations is split up between its outside shareholders and the other organization. The proportion that goes to the other organization is once again split up, and a fraction of it stays in the system. This amount is once more split up, and so on (Elliott et al, 2013).

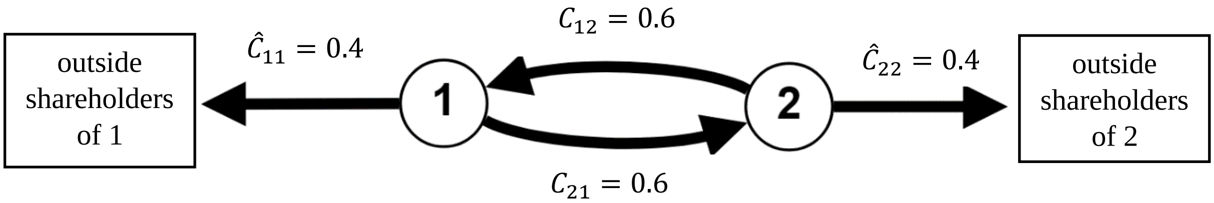

Figure 1: The network of cross-holdings as a weighted directed graph

The dependency matrix, $\boldsymbol{A}$, describes how the unit of income is ultimately divided between shareholders of the two organizations. While each of them directly owns 60% of the other, shareholders only receive $\frac{3}{8} = 37.5\%$ of the value generated by the other organization due to the presence of mutual cross-holding (Elliott et al, 2013).

Now, to easily understand how a negative shock to asset prices can cause a cascade of downgrades in the network, consider the following simplifications. Assume that $m = n$ and $\boldsymbol{D} = \boldsymbol{I}$, so that organization 1 exclusively owns asset 1 and organization 2 exclusively owns asset 2. In addition, suppose that there are three states of financial health. This means that organizations can have a rating of 1, 2 or 3 describing their creditworthiness from worst to best. One can think of the highest rating as investment grade, then junk grade, and finally default. With these assumptions, if initially both organizations have the highest rating, the equation for their market values simplifies to

$$\boldsymbol{v} = \boldsymbol{A}\left(\boldsymbol{p} - \sum_{l=1}^{2} \boldsymbol{b}^l(\boldsymbol{v})\right)$$

$$\Leftrightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \boldsymbol{A}\left(\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} - \begin{pmatrix} \beta_1^1 \mathbf{1}_{v_1 < \underline{v}_1^1} \\ \beta_2^1 \mathbf{1}_{v_2 < \underline{v}_2^1} \end{pmatrix} - \begin{pmatrix} \beta_1^2 \mathbf{1}_{v_1 < \underline{v}_1^2} \\ \beta_2^2 \mathbf{1}_{v_2 < \underline{v}_2^2} \end{pmatrix}\right).$$

Let the values for the two thresholds be $\underline{v}_i^2 = 60$ and $\underline{v}_i^1 = 20$ and the corresponding downgrade costs be $\beta_i^2 = 30$ and $\beta_i^1 = 20$ for both organizations $(i = 1, 2)$. Consider a solution to the above equation, in which both organizations are in investment grade, so that all indicators take the value 0. Refer to this as the initial equilibrium of the system. For example, let initial asset prices equal 100 an substitute into the equation to get

$$\boldsymbol{v}^{initial} = \boldsymbol{A}\boldsymbol{p}^{initial} = \begin{pmatrix} \frac{5}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{5}{8} \end{pmatrix} \begin{pmatrix} 100 \\ 100 \end{pmatrix} = \begin{pmatrix} 100 \\ 100 \end{pmatrix}.$$

These initial values are indeed equilibrium values, since they are above the threshold values for both organizations. Next, consider a negative shock that changes the prices of assets 1 and 2 from 100 to $p_1$and p_2, respectively, and thus changes the values of the organizations. The question of interest is what equilibrium will arise after the shock and what will the ratings be in this new equilibrium. Whether a cascade of downgrades emerges and what is the magnitude of a cascade greatly depends on the values of the parameters of the model. All of them and their potential effect on the outcome are detailed at the end of this section. However, the specific values of this example are a good illustration of how the network-based effects can lead to a cascade, while they still allow to demonstrate general behaviour in the model. In this case, one can derive the following conditions for the shock to result in a downgrade. If $v_1 = \frac{5}{8}p_1 + \frac{3}{8}p_2 < 60$, then organization 1 is downgraded to junk and incurs a cost of 30. If $v_1 = \frac{5}{8}p_1 + \frac{3}{8}p_2 < 20$, then it is downgraded to default and incurs an additional cost of 20. The boundary of the region defined by the first inequality is referred to as the junk boundary of organization 1 ($JB1$), while the second inequality defines its default boundary ($DB1$). Analogous conditions hold for organization 2 and the four boundaries can be illustrated as shown in Figure 2.1. If the shock is severe enough, so that the point of the new asset prices is below one of the boundaries, then a downgrade occurs. This is necessary to have different ratings in the new equilibrium compared to the initial one. For concreteness, consider an extreme shock specific to the asset of organization 1 such that asset 1 loses 90% of its value ($p_1 = 10$), while asset 2 is not impacted ($p_2 = 100$). Figure 2.1 also shows the first consequence of such a shock. Since the point $(10{,}100)$ is below the junk boundary of organization 1, it gets downgraded to junk grade. This is the immediate effect of the shock, which does not directly change the rating of organization 2. However, there are further indirect effects that can lead to a cascade of downgrades. Upon the downgrade to junk grade, organization 1 incurs a downgrade cost of $\beta_1^2 = 30$. Due to the cross-holdings, organization 2 bears a fraction of this and its value decreases by $A_{21} * \beta_1^2 = \frac{3}{8} * 30$. Consequently, the junk and default boundaries of organization 2 shift to the right. The new boundaries are given by the inequalities $\frac{3}{8}(p_1 - 30) + \frac{3}{8}p_2 < 60$ and $\frac{3}{8}(p_1 - 30) + \frac{3}{8}p_2 < 20$. These are referred to as $JB2'$ and $DB2'$ and are shown in Figure 2.2. It can be seen that the current asset prices $(10{,}100)$ are below the new junk boundary, so organization 2 gets downgraded to junk. This is the first step in a cascade of downgrades. Importantly, this downgrade is different from the first one of organization 1, in the sense that this is a network-based contagion effect, and not a direct effect of the shock. Without cross-holdings and downgrade costs, this second downgrade would not happen. At the following step, the default boundary of organization 1 shifts to the right ($DB1 \rightarrow DB1'$) due to the cost arising from 2's downgrade to junk grade. This is shown in Figure 2.3. In this step, organization 1 defaults and incurs a cost of 20, hence the default boundary of organization 2 shifts to the right again ($DB2' \rightarrow DB2''$). Figure 2.5 presents the final outcome and the new equilibrium. Organization 1 has defaulted, while organization 2 stays in junk grade, because its newest default boundary remains below the current asset prices. Thus, there are no more rating changes and an equilibrium is reached.

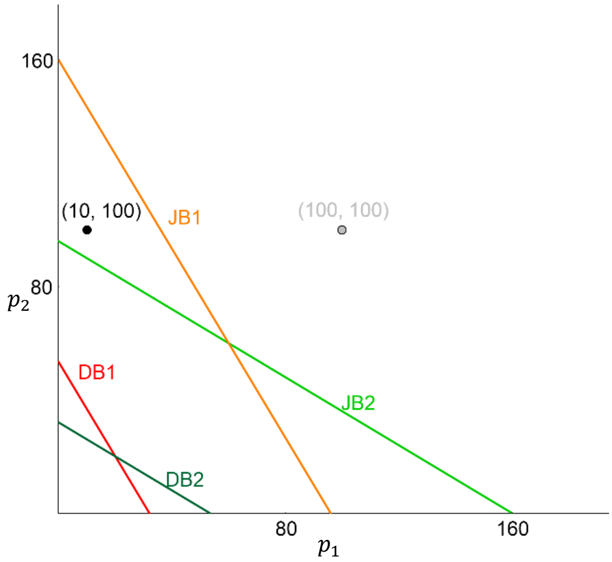

Figure 2.1: Junk and default boundaries

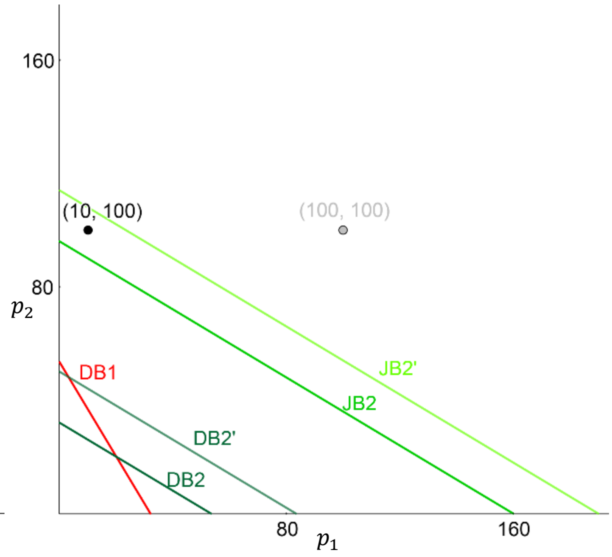

Figure 2.2: 1st downgrade due to contagion and the direct effect of a shock

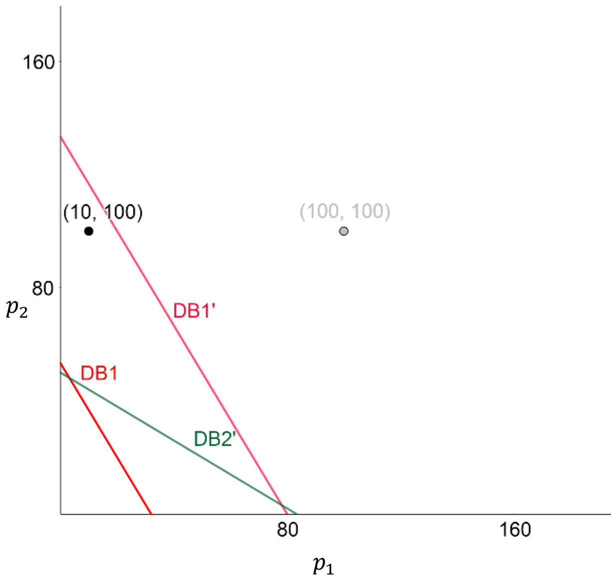

Figure 2.3: 2nd downgrade due to contagion

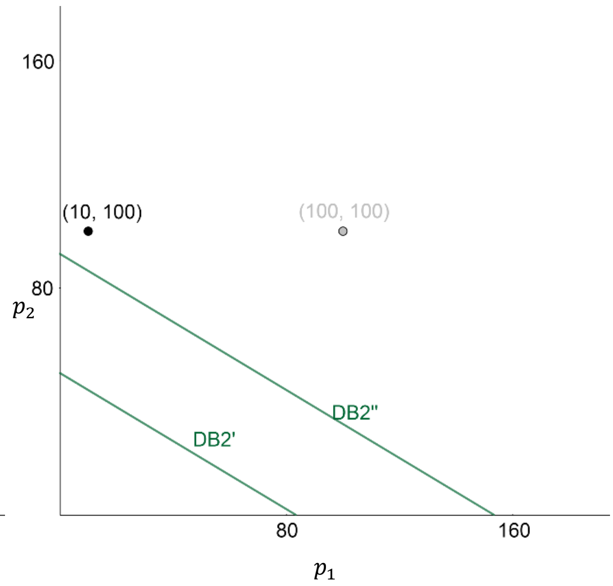

Figure 2.4: The new equilibrium

In this simple example, the figures make it easy to follow a cascade of downgrades step by step. However, as the dimensions (number of organizations and assets, number of states of creditworthiness) increase, it becomes impossible to depict the relevant boundaries as above. Nevertheless, a relatively simple algorithm can be applied to keep track of the rating changes caused by the initial shock and calculate the ratings in the resulting new equilibrium. The algorithm described below is based on the one used by Elliott et al (2014), but it is extended to accommodate the possibility of more than two states of creditworthiness of the organizations.

Let organizations have any ratings between 1 and s. In the simple example, both organizations had the highest possible rating in the initial equilibrium. However, the same method can be followed for any distribution of initial ratings, where organization i with an initial rating of q has thresholds $\underline{v}_i^1 < \cdots < \underline{v}_i^{q-1}$ and downgrade costs $\beta_i^1, \ldots, \beta_i^{q-1}$. Denote the vector of asset prices after the shock by $\boldsymbol{p}$. Let $\widetilde{\boldsymbol{b}}$ be the n x 1 vector of downgrade costs in the beginning of a given step of the algorithm. Initialize $\widetilde{\boldsymbol{b}} = \boldsymbol{0}$ for the first step. At each step of the algorithm:

I. Let r_i be the rating of organization i at the beginning of the step
II. Calculate the vector of values as $\boldsymbol{A}(\boldsymbol{Dp} - \widetilde{\boldsymbol{b}})$.
III. Compare values to thresholds: for each i, find the lowest rating of l for which $\left[\boldsymbol{A}(\boldsymbol{Dp} - \widetilde{\boldsymbol{b}})\right]_i < \underline{v}_i^l$.
IV. If such rating does not exist, then organization i keeps the same rating as at the beginning of the step, otherwise, it is downgraded to a rating of l.
V. Calculate downgrade costs:
    a. $\left[\widetilde{\boldsymbol{b}}\right]_i = \sum_{k=l}^{r_i - 1} \beta_i^k$ if i has been downgraded to l, and 0 otherwise.
VI. If there are no changes in the rating of any of the organizations, then the algorithm terminates, otherwise return to I.

This algorithm concludes in a finite number of steps and allows one to keep track of the ratings step by step. The ratings at the end correspond to the new equilibrium that emerges. The first step captures the direct effect of the shock, while any downgrades in later steps are due to network contagion. One can quantify the likelihood and extent of a cascade by, for example, calculating the total number of downgrades in a given network of organizations. Regardless of the main variable of interest, this framework allows for comparative statics in the model. The next subsection describes the parameters and their potential effects on the final outcome and highlights the ones that are related to the main questions of the thesis.

2.4 The parameters of the model

The first parameter of the model is the network structure, captured by the dependency matrix. As mentioned before, it is the focus of many papers in the literature on financial contagion in networks. It is also one of the parameters of interest of this thesis, here in the context of network contagion in credit rating migration. Elliott et al (2014) characterize the network of cross-holdings by two of its attributes. Since the network structure is modelled identically in this thesis by the dependency matrix, the same characterization is used here. The first attribute is diversification. It captures how spread out the cross-holdings are by measuring the average number of cross-holders of an organization. The second attribute is integration. It captures how deep the cross-holdings of an organization are by quantifying the proportion of an organization that is held by other organizations (Elliott et al, 2013). These two properties allow one to investigate how the contagion effects captured by the model change for different structures of the network. The next section introduces the precise definition of diversification and integration that help to simulate the cascade of downgrades for a parametric family of network structures.

The second parameter is the distribution of the ratings in the initial equilibrium. The modification of the original model presented previously in this section has the innovation that the algorithm can be calculated for different initial ratings. The question of interest is how susceptible a network with more good initial ratings (e.g. investment grade) to contagion is compared to one with a higher number of bad ratings (e.g. junk grade). This feature is also in line with the fact that in reality, not all organizations of an interdependent network have the same creditworthiness. The second main focus of the thesis is to assess the effect of this parameter and the exact parametric distribution used in the simulation of the model is described in the upcoming section.

The third parameter of the model is the size of the initial shock. Holding all else constant, a larger negative shock may cause more downgrades in the first step, hence more downgrade costs might be incurred in the network and thus the subsequent contagion can be greater. Importantly, the shock has to be severe enough that it results in at least one downgrade directly, otherwise a cascade cannot emerge. The last two parameters are the threshold values and the downgrade costs. The way in which their different values affect the outcome is straightforward, still it could be an interesting topic to study their effect for various distributions of the two jointly and separately. For example, in reality, a single-notch drop from AA+ to AA may be associated with a lower relative increase in the cost of capital than a single-notch drop from B- to CCC, and thus may be less likely to set off a contagion. Nevertheless, these parameters are not the focus of this thesis, therefore the main results are derived using a parsimonious parametrization for them.

3. SIMULATION OF THE MODEL AND COMPARATIVE STATICS

Detailed cross-holdings data on the level of individual organizations are rarely available. Therefore, simulation using random graphs is a widely applied method to model financial networks in an attempt to gain insight into the key mechanisms. This section describes a possible technique that can capture a family of network structures characterized by the two parameters mentioned in the previous section, diversification and integration. It also shows how the distribution of the initial ratings might be incorporated in simulations and how a cascade of downgrades can be simulated in the model using the algorithm specified earlier. Finally, the main results of the thesis are presented that illustrate the effect of the parameters of interest on downgrade contagion.

3.1 Random network structure, diversification and integration

In order to carry out simulation, one has to find a way to parameterize the network formation which involves defining the matrix of cross-holdings, $\boldsymbol{C}$, and subsequently the dependency matrix. As mentioned before, Elliott et al (2014) propose diversification and integration as two parameters that can characterize the network structure. They also suggest how randomness can be incorporated, so that simulation experiments can be performed. Fix the number of organizations n and consider the n x n matrix $\boldsymbol{G}$, for which $G_{ij} = 1$, if organization i has cross-holdings in organization j, and $G_{ij} = 0$, otherwise. $\boldsymbol{G}$ is called the adjacency matrix of an unweighted, directed graph. One way to make this a random graph is to let, for $i \neq j$, $G_{ij} = 1$ with probability $\frac{d}{(n-1)}$ and $G_{ij} = 0$ with probability $1 - \frac{d}{(n-1)}$, where d is a real parameter between 0 and $n - 1$. Essentially, for $i \neq j$, G_{ij} are i.i.d. random variables of Bernoulli distribution with probability $\frac{d}{(n-1)}$. To be consistent with the model, let $G_{ii} = 0$ for all i. This construction ensures that the expected out-degree of any node i is equal to:

$$E[d_i^{out}] = E\left[\sum_{j=1}^{n} G_{ji}\right] = \sum_{j=1}^{n} E\left[G_{ji}\right] = (n-1) * \frac{d}{(n-1)} = d$$

The out-degree gives the number of edges pointing outwards from a node, which is equivalent to the number of cross-holders an organization has in the present context of the model. Therefore, it is a measure of how spread out or how diversified the cross-holdings are.

Hence, the random matrix $\boldsymbol{G}$ defined above can capture the extent of diversification in the network structure through the single parameter d (Elliott et al, 2013).

To eventually construct the matrix $\boldsymbol{C}$, one has to assign weights to the edges of the random graph represented by $\boldsymbol{G}$. A possible way to do this is to suppose that a fraction c of each organization i is owned by others, distributed evenly among the d_i^{out} number of cross-holders of i. This means a weight of $\frac{c}{d_i^{out}}$ for all edges pointing outwards from the node i. The remaining $1-c$ fraction is the part that is owned by outside shareholders of i, so that $\hat{C}_{ii} = 1-c$ for all i. Precisely, for a given realization of the random matrix $\boldsymbol{G}$, the matrix of cross-holdings $\boldsymbol{C}$, can be calculated as

$$C_{ij} = \begin{cases} \dfrac{c * G_{ij}}{d_j^{out}}, & if\ d_j^{out} > 0 \\ 0, & otherwise. \end{cases}$$

For all i, $G_{ii} = 0$ implies that $C_{ii} = 0$, which is once again consistent with the assumptions of the model. The value of c determines the portion of an organization that is held by its cross-holders and by its external shareholders. Therefore, this parameter measures how deep or how integrated cross-holdings are. Once $\boldsymbol{C}$ is defined, $\widehat{\boldsymbol{C}}$ and the dependency matrix can be computed. Hence, the random network structure is completely characterized by the two parameters of diversification and integration. Using many realizations, one can simulate cascades of failures with the algorithm introduced before and investigate how the total number of downgrades depend on the values of d and c. On one hand, holding integration constant, a higher value of diversification means that on average, the fraction c of each organization is divided among a higher number of other organizations with each of them owning a smaller part. On the other hand, for a realization of $\boldsymbol{G}$**,** as integration increases, outside shareholders hold less of each organization, while each organization owns a larger part of the other organizations in which it has cross-holdings (Elliott et al, 2013). The main results presented later in this section show the effects of diversification and integration on the extent downgrade contagion separately for the two parameters.

3.2 The distribution of the initial ratings

The parametrization of random network formation using diversification and integration is borrowed from Elliott et al (2014). The novelty of this thesis is to introduce rating downgrades other than default, which allows for 2 new types of analysis. Firstly, one can study the equilibrium behaviour of non-default downgrades which in reality are much more frequent.

Secondly, the model of this thesis allows one to study how the network effects of a shock change based on the distribution of organizations' ratings prior to the shock. The following simple method can be considered to simulate this effect with the addition of a single extra parameter to the model. Take the network structure and the other parameters (shock size, threshold values and downgrade costs) as given. For simplicity, assume that there are three states of creditworthiness - investment grade, junk grade and default. In the initial equilibrium organizations can be either in investment grade or junk grade. Since the model and the algorithm is designed to capture cascades of downgrades, the possibility of an initial rating of default is excluded, although it might be the case in some real networks. In order to get a clean sense of the modelled mechanisms, I make a simple assumption that a one parameter distribution can describe the initial ratings. Suppose that each organization can start as investment grade with probability q and as junk grade with probability $1-q$. This way, one can perform comparative statics with respect to the single parameter q.

The proposed method restricts the simulation of the model to only three different ratings of the organizations and a simple family of distributions for the ratings of the initial equilibrium. Other distributions with potentially more parameters can be considered in the case of more ratings and can be calibrated using observed rating distributions for a given portfolio. Having more than three states is arguably more realistic, however, the important distinction of the model used in this thesis compared to the two-state model, comes from not having all organizations start from the same state. Further increasing the number of states using a more involved parametrization may theoretically provide additional insights at the expense of extra complexity, however the proposed parsimonious parametrization should be sufficient to flush out the substantial novelty and the mechanisms of the model.

3.3 Simulations of cascades of downgrades

Before turning to the main results of the thesis, this subsection collects all of the components that are required to run simulations. The algorithm described in the previous section allows one to calculate the number of downgrades and the final rating of each organization in the new equilibrium that emerges as a result of an initial shock, for a given set of parameter values. The values used in the simulations are as follows.

The network structure is given by the number of organizations $n \geq 2$, the parameter of diversification $0 \leq d \leq n-1$, and the parameter of integration $0 \leq c \leq 1$. The initial ratings are determined by the probability $0 \leq q \leq 1$, as described previously.

The other parameters of the model are not the main focus of the thesis, hence their values are fixed in a simple way consistent with Elliott et al (2014). Similar to previous sections, it is assumed that $m = n$ and $\boldsymbol{D} = \boldsymbol{I}$, so that each organization fully owns its proprietary asset. Prior to the shock, all asset prices are equal to 1. This determines the values in the initial equilibrium as $\boldsymbol{v}^{initial} = \boldsymbol{A}\boldsymbol{p}^{initial}$. If organization i starts in junk grade initially, then it has one threshold value $\underline{v}_i^1$. This is set as a fixed percentage of i's initial value, that is

$$\underline{v}_i^1 = \theta * v_i^{initial}$$

, where $0 < \theta < 1$. Essentially, a $(1 - \theta) * 100$ percentage decrease of the initial value results in a default. Instead, if i has an initial rating of investment grade, then it has an additional threshold value $\underline{v}_i^2$, defined as:

$$\underline{v}_i^1 < \underline{v}_i^2 = \left[\theta + \frac{1}{2}(1 - \theta)\right] * v_i^{initial}$$

This means that the higher threshold is equidistant from the lower threshold and the initial value of i. Half the size of a decrease of the initial value is required to get downgraded to junk compared to default. As a robustness check, this assumption is relaxed and the simulations are also performed for the case when the higher threshold is further away from the lower, making a first downgrade more likely than an eventual default, which appears a more realistic parametrization. The corresponding failure costs are set to be $\beta_i^1 = \frac{1}{2} * \underline{v}_i^1$ and $\beta_i^2 = \frac{1}{2} * (\underline{v}_i^2 - \underline{v}_i^1)$ for all simulations. Different specifications could also be considered here, but instead, these are held fixed to be able to focus on the parameters of interest. The final ingredient needed to perform a simulation is to set the size of the shock. It is modelled as an idiosyncratic shock that hits a single organization chosen uniformly at random, whose proprietary asset loses all its value, meaning that the asset's price drops to 0 (Elliott et al, 2013). This extreme shock with some sufficiently high value for θ ensures that at least one downgrade occurs directly after the shock.

Now, this setup is suitable to use the algorithm described in the previous section to simulate the potential cascades of downgrades that emerge as a result of the shock. After taking many realizations of the random network structure and the initial ratings, one can calculate the average number of downgrades for each initial rating state. Then, it is also straight forward to implement comparative statics and investigate the effects of the parameters of interest.

3.4 The effect of diversification and integration

When simulating the two-state model, Elliott et al (2014) find that diversification has a nonlinear effect on the number of organizations that fail in the new equilibrium after the shock. For low values of the expected out-degree ($d \leq 1.5$), the proportion of failing organizations stays relatively small. The intuition is that the network is not connected enough for contagion to reach many nodes in the network. For medium levels of diversification ($2 \leq d \leq 6$), the initial shock and subsequent failures are able to propagate through the network and affect a large number of organizations resulting in further failures. But when d is sufficiently large ($8 \leq d$), the fraction that each organization cross-holds in others is so low that organizations become insensitive to the failure of others and the initial shock cannot cause a cascade.

Integration in Elliott et al (2014) exhibits a different nonlinearity. As c increases from 0 to 0.5, the number of failures strictly increase as organizations become more reliant on others. However, for high levels of integration, organizations hold a larger part of other organizations and depend less on the value of their own proprietary asset. Therefore, above a threshold for c, the initial shock does not cause a direct failure and thus a cascade cannot emerge. The first task of this thesis is to investigate the behaviour of these two parameters in the case when organizations can have three different states of creditworthiness and not all organizations start in the same state.

The simulations in this thesis are performed for a network of 50 organizations and first, for the equidistant thresholds. In order to investigate the effect of different levels of diversification, integration is fixed at $c = 0.5$ and the probability of any organization starting in investment grade at $q = 0.5$. Initially, the outcome variable of interest is the total number of downgrades of all organizations irrespective of their initial rating. This includes downgrades to junk grade and defaults as well. Figure 3 shows the outcome as a function of the expected out-degree (d) for different thresholds. The parameter of diversification takes values between 0 and 49 with a step size of 0.3. For each value of d, the differences between the initial ratings and the final ratings are summed up across all organizations and then averaged over 500 simulations. The results reinforce the findings of Elliott et al (2014). The nonlinear effect of diversification is confirmed for all values of θ, where higher thresholds lead to a more severe cascade with more downgrades. The exact values of d that gives the sweet spot for a cascade to emerge depend on θ, but generally referred to as a medium level of diversification.

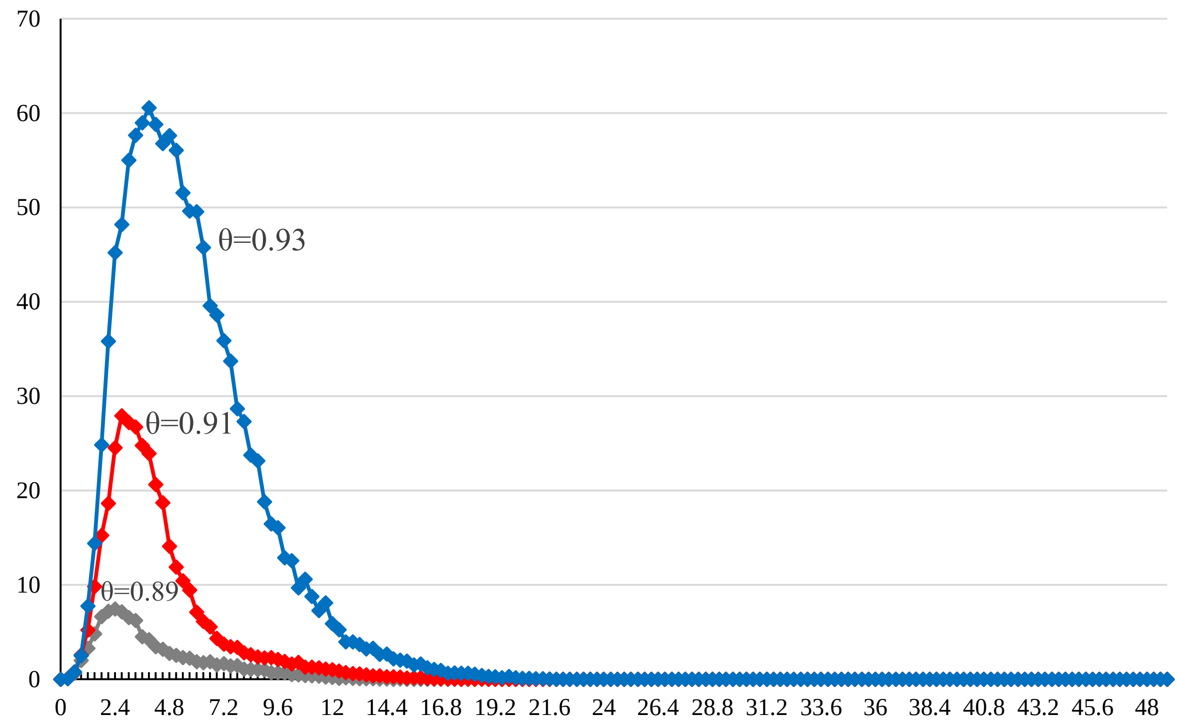

Figure 3: The effect of diversification on the extent of the cascade of downgrades for different values of θ (n=50, c=0.5, q=0.5, 500 simulations)

Since not all organizations start with the same rating, the three-state model allows one to plot the total number of downgrades as a sum of the downgrades from investment grade (IG) and from junk grade (JG). The value of $q = 0.5$ means that on average, 25 organizations start in each initial category. Figure 4 shows the decomposed effect of diversification for $\theta = 0.93$. The curves continue to exhibit the same nonlinearity, however, as the total number of downgrades decline for $d > 12$, the downgrades from junk disappear first and a small cascade remains only consisting of downgrades of those organizations that started in investment grade. Essentially, as the network becomes highly diversified and the extent of the cascade decreases overall, the organizations that start with the worse rating are saved from default first, and the organizations that have the higher initial rating only fully avoid downgrades if diversification increases even more.

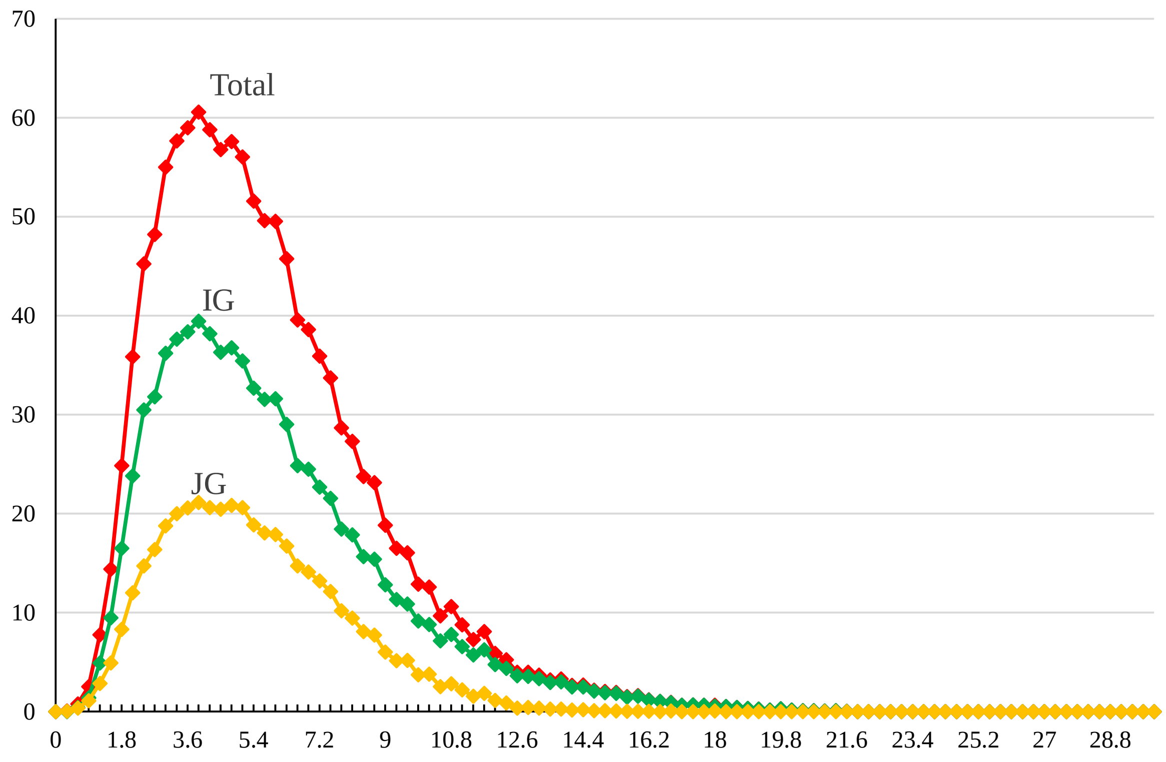

Figure 4: The effect of diversification decomposed by initial ratings: all downgrades (red), downgrades from IG (green), downgrades from junk (yellow)
(θ=0.93, n=50, c=0.5, q=0.5, 500 simulations)

To fully understand the process, the downgrades corresponding to the green curve in Figure 4 can be further split up: some organizations that start in investment grade are downgraded twice and end up in default, while some of them only suffer one downgrade and stay in junk in the new equilibrium. Figure 5 shows the total number of downgrades decomposed to the three different types: downgrades from investment grade to default (Type I), from investment grade to junk grade (Type II) and from junk grade to default (Type III). As diversification changes from low to medium levels ($0 \leq d \leq 4$), almost all downgrades are of Type I and Type III, illustrating that most organizations which are affected by the cascade eventually end up in default. As d increases further, the decrease in the number of defaults (Type I + Type III) is the main contributor to the decrease of the total number of downgrades, however, the number of Type II downgrades slightly increases. Both Type I and Type III downgrades disappear when diversification reaches a sufficiently high value ($d \approx 13$) and here all of the remaining downgrades are of Type II. With this observation, the intuition underlying the mechanism of increasing diversification and its effect on the cascade of downgrades can be revised in the following way. For low levels of diversification, a cascade cannot emerge as the network is not connected enough but, if diversification increases to medium levels, the shock can reach many organizations and cause a large number of downgrades. In fact, most of the affected organizations end up in default according to these simulations. For high levels of diversification the cascade disappears as negative effects are spread out among a large number of organizations. These findings confirm the results of the two-state model on the effect of diversification and the sweet spot for cascades. The additional insight of the three-state model is that there is a region, where diversification is high enough so that a cascade of defaults does not emerge, but not sufficiently high to prevent some initially higher rated organizations from suffering a single downgrade of their credit rating in a smaller cascade. In real-world terms this means that there exists a second sweet spot for diversification where in response to a shock only a cascade of downgrades emerges but not a cascade of defaults which tends to be a feature of real networks where defaults are relatively rare.

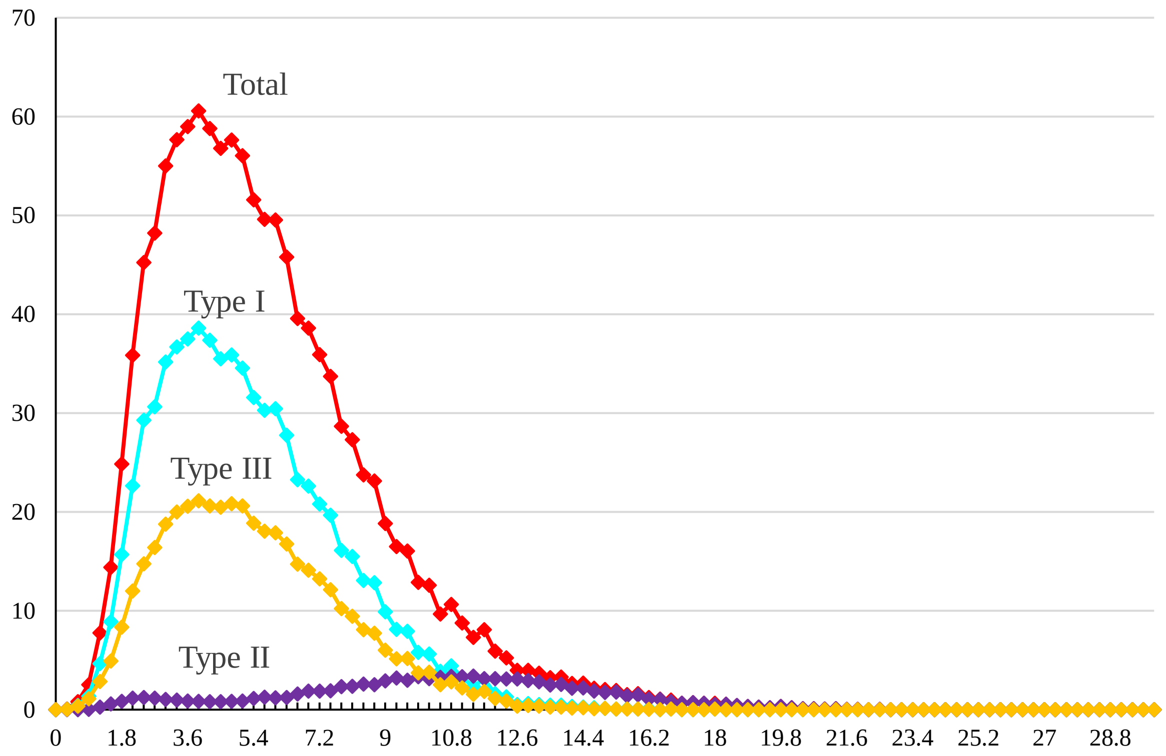

Figure 5: The effect of diversification split up to downgrades from IG to default (Type I), from IG to JG (Type II) and from JG to default (Type III) (θ=0.93, n=50, c=0.5, q=0.5, 500 simulations)

So far, the results were based on simulations that were performed for the case of equidistant thresholds. The figures showed that when a large cascade emerges, most of the downgrades lead to the default of the affected organizations. In order to check the robustness of the results and possibly get a better illustration of the previously described mechanism, one can modify the thresholds to be asymmetric: for organizations of the higher initial rating, the distance between the two thresholds can be increased to make Type II downgrades more likely. This modification is also consistent with the intuition that in reality, defaults are rare and thus should be less frequent than single-rating drops from a high credit rating. For this purpose, the distance between the thresholds are modified as

$$\underline{v}_i^1 = \theta * v_i^{initial}$$

$$\underline{v}_i^2 = \left[\theta + \frac{3}{4}(1-\theta)\right] * v_i^{initial}$$

, if i has an initial rating of investment grade. Figure 6 shows the effect of diversification based on simulations using the modified thresholds for different values of θ. Besides the already observed effect of diversification, the curves show a second source of nonlinearity. Previously, the cascade died out as the network became highly diversified, but now, there is a new region of high diversification with an additional significant cascade. Before the downgrades fully fade after the first sweet spot, a second hump develops which is lower than the first one but wider, meaning that it only disappears for an additional, even higher increase in diversification. The decomposition of the total number of downgrades to the three different types can provide the explanation for this new result.

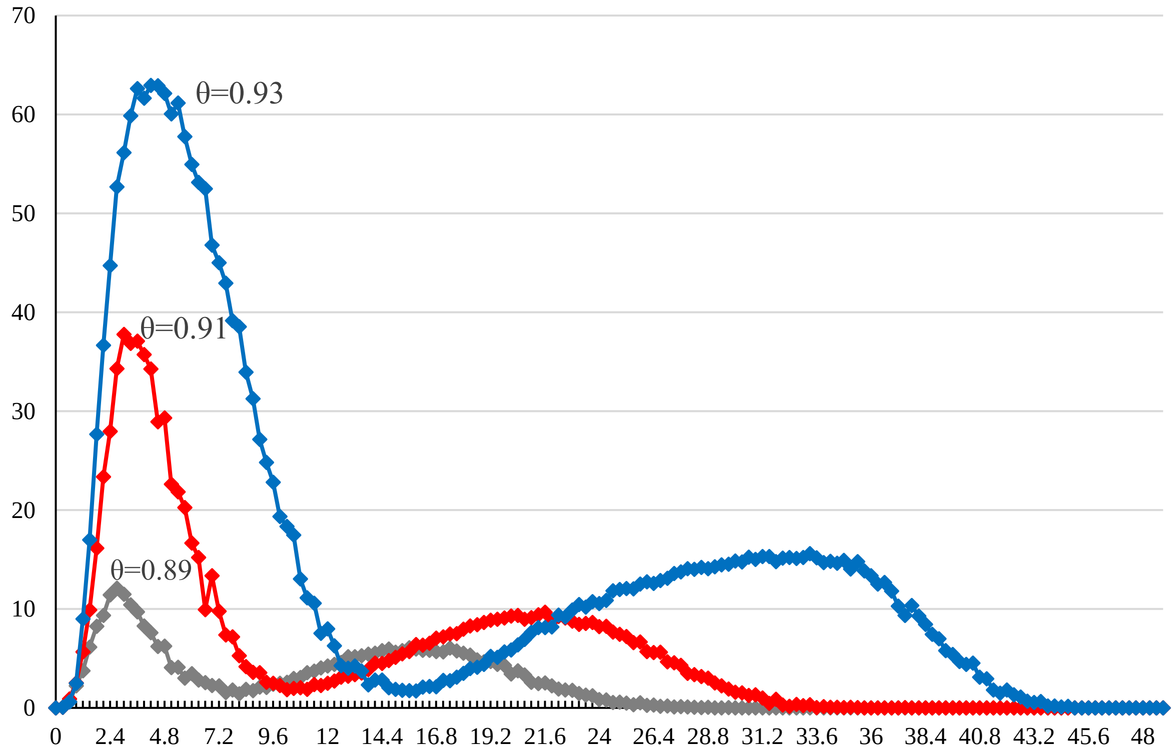

Figure 6: The effect of diversification for the modified thresholds and different values of θ (n=50, c=0.5, q=0.5, 500 simulations)

Figure 7 shows the three types of downgrades and their sum for $\theta = 0.91$ in the case of the modified thresholds. In the region of $0 \leq d \leq 4$, Type II downgrades are somewhat more frequent than previously, as expected. But still, the majority of organizations affected in the cascade end up in default. An even greater asymmetry of the thresholds would be required to further increase the balance the types of downgrades in this region. As diversification increases over the first sweet spot, all three types of downgrades decrease. Eventually, types I and III disappear, so there are virtually no defaults. However, the Type II downgrades start two increase before the cascade dies out. The second hump entirely consists of downgrades from investment grade to junk which is ultimately an effect of the modified thresholds that made this type more likely. These observations lead to the following conclusion in the case when a smaller decrease in value is enough to suffer a single-rating downgrade, while defaulting is relatively harder for an organization of a high initial rating. The same mechanism of increasing diversification holds for values around the right end of the first sweet spot as before: defaults disappear from the cascade first while downgrades from investment grade to junk still remain. However, these do not die out quickly for further diversification of the network, but their number increases, causing a more prominent second sweet spot for a cascade of this type of downgrades that is more severe and can potentially emerge for a wide range of diversification. Thus, this case reinforces and further illustrates how a more realistic cascade of downgrades can emerge. Further asymmetry of the thresholds lead to a greater number of Type II downgrades overall and the second sweet spot stands out even more. Nevertheless, these additional simulations do not provide any new insights or better illustrations and so are excluded from the thesis but are available upon request.

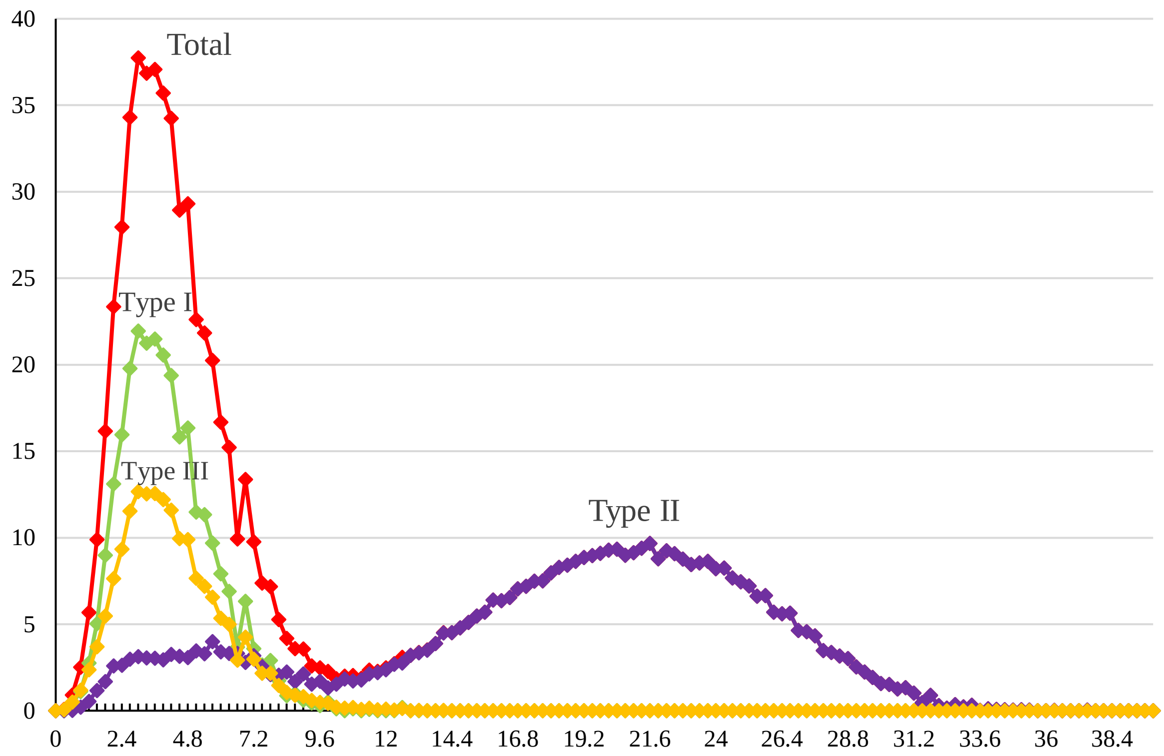

Figure 7: The effect of diversification for the three types of downgrades for the modified thresholds (θ=0.91, n=50, c=0.5, q=0.5, 500 simulations)

Turning to integration, Figure 7 shows the total number of downgrades as a function of expected out-degree (d) for different levels of integration using equidistant thresholds fixed at $\theta = 0.91$. It can be seen that as c increases from 0.1 to 0.6, the magnitude of the cascade of downgrades increase in a diminishing way. When increasing integration further, eventually there comes a value of c above which the number of downgrades drop to approximately 0, because the shock can almost never cause a downgrade directly. This threshold of c depends on the value of θ and in particular, the distance between the highest threshold and the initial value of the organization that is hit by the shock directly. This nonlinearity of integration is analogous to the previously mentioned effect in the two-state model, so one can think of it as a sanity check to demonstrate that no important mechanisms from the 2-state model have been interrupted.

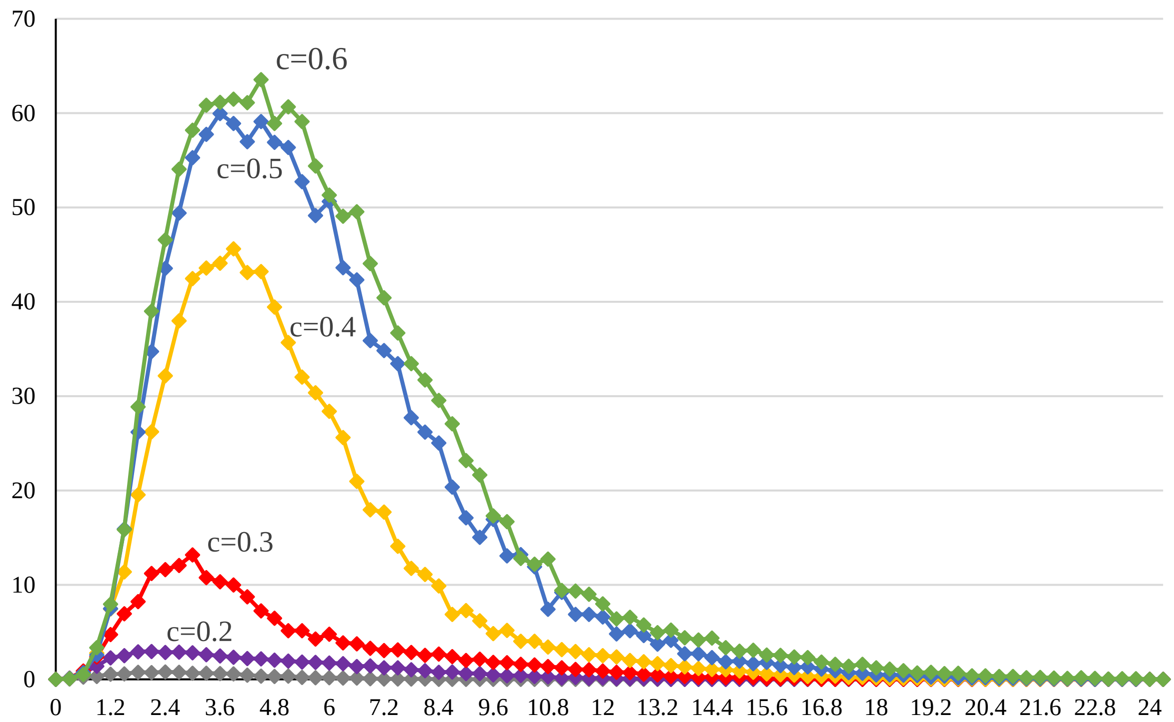

Figure 7: The effect of integration: the number of downgrades as a function of diversification for different levels of integration (θ=0.91, n=50, q=0.5, 500 simulations)

3.5 The effect of the distribution of the initial ratings

The next main task of the thesis is to assess the effect of the new parameter of the three-state model: the probability of an organization having a high initial rating (i.e. IG). The value of q is varied between 0 and 1 with a step size 0.1. The boundary values correspond to the two-state model as special case which has already been covered in the literature. The equidistant thresholds are defined by $\theta = 0.93$ and integration is fixed at $c = 0.5$. The three different types of downgrades are tracked as a percentage of the organizations of each initial rating and as a function of d. The reason for focusing on percentages rather than absolute numbers is that changing q necessarily changes the number of organizations potentially subject to each type of downgrade. Figure 8 shows the proportion of defaulting organizations for an initial rating of junk grade (Type III). As q increases the extent of the cascade of these downgrades increases, but in a decreasing way. This means that in a network where the majority of the organizations have a bad rating (low q), a smaller fraction of these organizations will default in a cascade compared to a network in which organizations of the bad rating are the minority (high q). The intuition behind this is that in a network with a large number of organizations of a high rating, a potential cascade involves more downgrades and larger cumulative downgrade costs across the economy which have a more severe effect on the value of all organizations, leading to more downgrades for bad organizations as well.

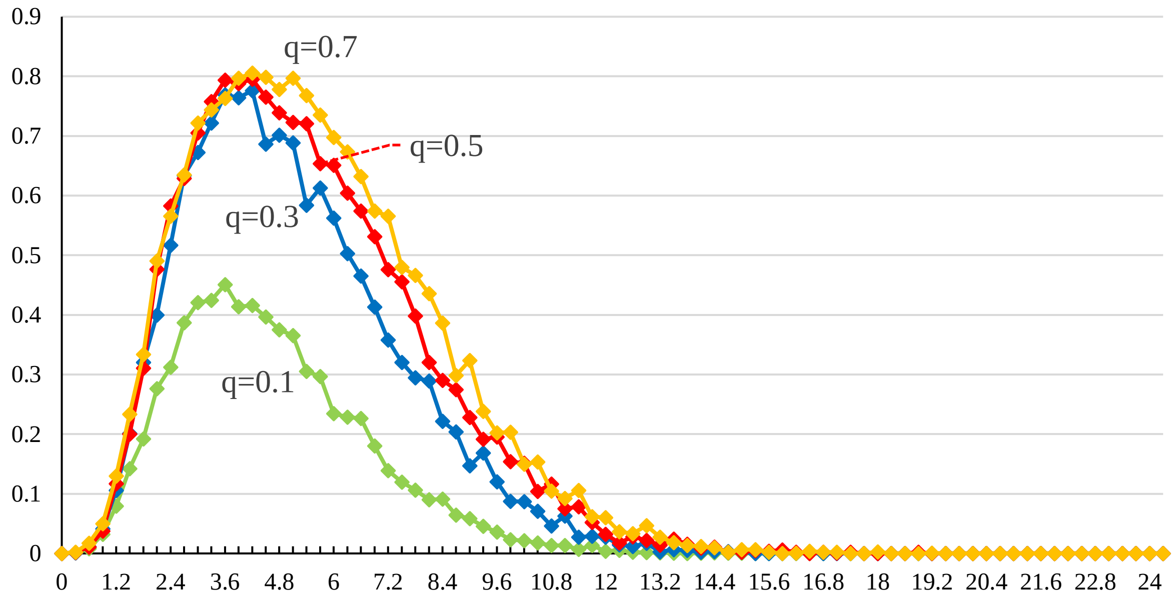

Figure 8: The effect of the probability of a high initial rating: the percentage of defaulting organizations for an initial rating of junk as a function of d, for different values of q (θ=0.93, n=50, c=0.5, 500 simulations)

This finding is reinforced when looking at Type I downgrades. Figure 9 shows the share of organizations with an initial rating of investment grade that default in a cascade for a low and a high value of q. The effect of the average number of good and bad organizations in the network is the same as in the previous case. The figure also shows the other fraction of organizations that corresponds to Type II downgrades for the two values of q. In this case, in the first sweet spot of medium diversification, the effect is opposite as before: a higher q means a downgrade of this type is less likely. However, this actually confirms the previous intuition, since when q is high and a large, severe cascade emerges, most of the affected organizations do not stop at one downgrade, but suffer two and end up in default. Besides, in the second sweet spot of high diversification, when only a smaller cascade of Type II downgrades develops, a network with more organizations of high initial rating slightly increases the number of downgrades in the cascade, but this is not significant as the cascade is already small and a large cost of downgrades cannot accumulate. Essentially, when the ingredients of medium levels of diversification and integration are present, in a network consisting of mostly good organizations, the emerging cascade of downgrades is more severe and leads to more defaults than in a network of mostly bad organizations. In addition, if diversification is such that a more realistic cascade of single-rating downgrades of good organizations develops, then the extent of this increases in the average number of good organizations in the network, but not significantly, as the number of downgrades in this cascade is small in the second sweet spot.

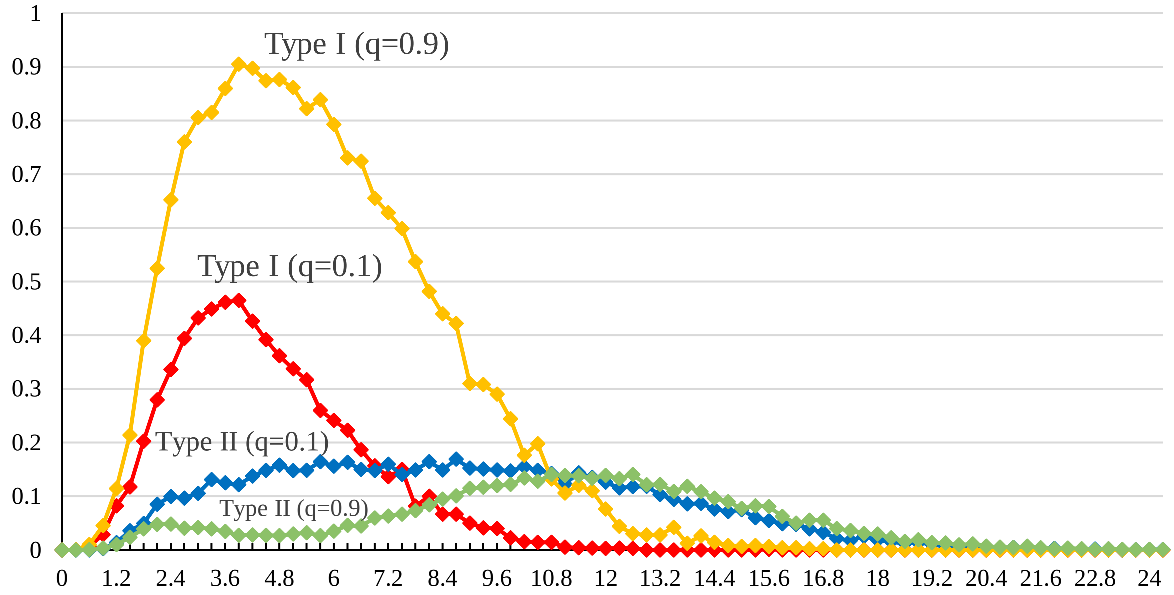

Figure 9: The effect of the probability of a high initial rating: the percentage of the two types of downgraded organizations for an initial rating of IG as a function of d for different values of q (θ=0.93, n=50, c=0.5, 500 simulations)

Modifying the thresholds in exactly the same way as before to make Type II downgrades more likely may illustrate this effect better. Figure 10 shows the analogous curves as Figure 9 for $\theta = 0.91$, but obtained from simulations using the modified thresholds. Indeed, it confirms the previous observations, but the effect of q is more prominent here in the region of the second sweet spot, as the cascade of Type II downgrades is greater in magnitude.

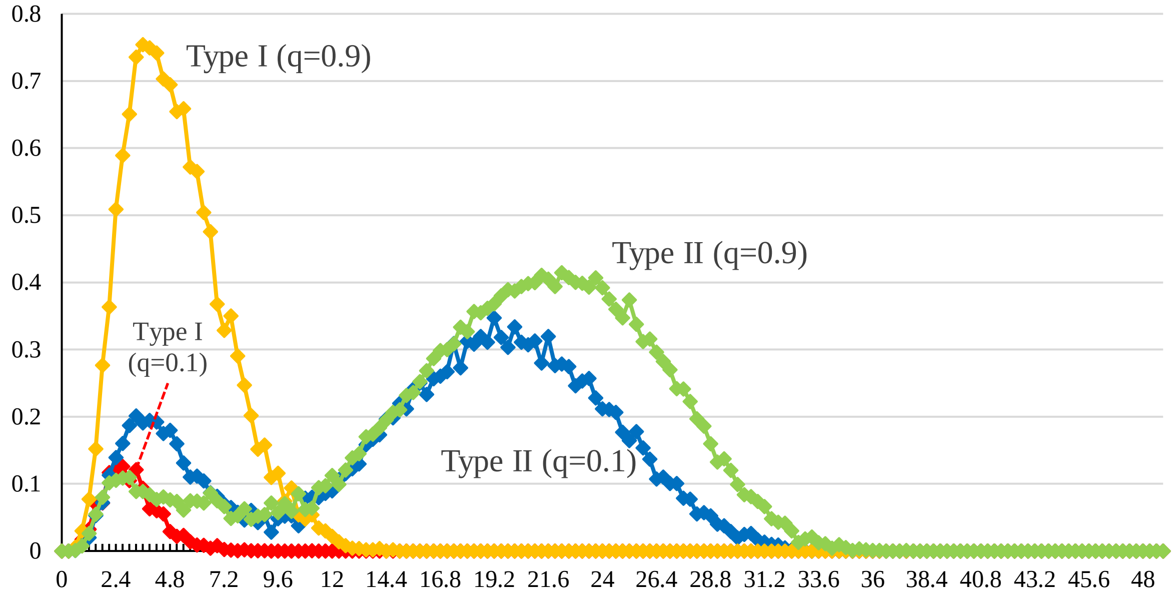

Figure 10 The effect of the probability of a high initial rating: frequency of Type I and Type II downgrades for the modified thresholds (θ=0.91, n=50, c=0.5, 500 simulations)

4. ILLUSTRATION OF THE MODEL USING REAL DATA

Although real-world data on financial networks is seldom available, there are other methods besides simulation that can be used to study contagion. In particular, the true underlying network of a set of organizations may be estimated by a proxy-network that describes a similar dependence as the original. This section presents an example of how real data on international trade can be used to approximate the network of cross-holdings among a large number of countries around the world. It also introduces some possible estimates for the values of the parameters of the model that allow implementation and, based on actual sovereign credit ratings, prediction on how these might change as a result of a negative macroeconomic shock.

4.1 Financial contagion and the trade network

A suitable proxy to the network of cross-holdings has to capture a similar interaction among the members of the network, but with the additional advantage of the availability of detailed, standardized data. Kali and Reyes (2010) argue that the international trade network is an appropriate candidate to study the network-based effects of financial contagion. In particular, they investigate whether the properties of a network of countries constructed based on country-level exports can explain stock market performance during financial crises. They find that the level of connectedness of a country in the trade network significantly affects the magnitude of the adverse effects of a negative shock and a subsequent crisis. Essentially, their findings support the notion that the network structure of international trades is suitable to analyse the propagation of financial contagion across the world. When one wants to isolate network-effects, it is hard to distinguish the relative significance of trade linkages and the actual financial flows between a pair of countries, because usually these are present simultaneously. Nevertheless, the trade relationships can be considered as the catalyst for the various financial flows, such as trade credit and international loans. In the context of the model of this thesis, the true network may be based on the consolidated cross-holdings of debt among the financial sectors of the countries for which detailed data is relatively hard to come by. In this framework, the value of a debtor depends on the borrower's ability to repay its obligations. In the trade network, the revenue of exporting countries comes from the importing ones, therefore the former depend on the latter (Kali and Reyes, 2005). Thus, data on international exports can be used to construct the dependency matrix, which defines the network structure in the model. The following subsections describes how the model might be applied to real data, how each of its parameters can be calibrated, and how the framework can be used to make a prediction for the downgrades of sovereigns following a shock.

4.2 The data

To implement the model, one has to make assumptions about the potential estimates of the values of the parameters. These assumptions and possible sources of data are described below for each parameter.

First, the network structure is approximated by the trade network for the reasons mentioned previously. The export data used in this thesis is from the The United Nations Commodity Trade Statistics Database (UN Comtrade). Bilateral export amounts are available at the end of each year for most countries across the world. Table 1 presents a portion of the trade matrix for the year 2015 as an example. It gives the raw amounts of export for Brazil (BRA) and the seven other countries that depend on Brazilian import the most: Argentina (ARG), Bolivia (BOL), Chile (CHL), Algeria (DZA), Nigeria (NGA), Paraguay (PRY) and Uruguay (URY). These are the countries for which the share of their export to Brazil is the highest across the world. This set of countries is specifically picked to here as the illustration of the model will be based on this component of the trade network. The rows represent exporting countries while the columns represent the importing ones. For example, Brazilian export to Bolivia was around $1.480M in 2015, while the value of goods exported from Bolivia to Brazil was about $2.447M.

	ARG	BOL	BRA	CHL	DZA	NGA	PRY	URY
ARG	0	621,672	10,097,998	2,404,404	1,147,789	88,762	1,054,580	1,232,187
BOL	1,473,716	0	2,447,485	82,699	0	0	32,027	6,045
BRA	12,777,105	1,480,823	0	3,968,371	992,883	687,818	2,472,582	2,726,121
CHL	800,612	1,286,000	3,047,340	0	3992	30,685	552,641	149,160
DZA	0	0	1,481,833	3,124	0	443	0	3,275
NGA	437,112	1	3,184,504	757	971	0	0	283,156
PRY	570,862	69,158	2,622,626	591,118	45,324	23,810	0	151,897
URY	380,340	35,238	1,134,192	117,428	73,860	24,957	116,428	0

Table 1: 2015 nominal export values for a part of the trade network in 1000 USD

The complete trade matrix consists of 170 countries and it provides the estimate for the actual cross-holdings of debt in the network. To convert these dollar values to the matrix $\boldsymbol{C}$ which has entries between 0 and 1, the fraction of each country that is held by its cross-holders is assumed to be $1/3$, based on the estimate in Elliott et al (2014). This means that $\boldsymbol{C}$ can be calculated by summing up the raw export values in each column and dividing the entries of each column by 3 times the corresponding column sum. This assumption also determines that $\hat{C}_{ii} = 2/3$ for all i and allows one to calculate the dependency matrix using the formula $\boldsymbol{A} = \widehat{\boldsymbol{C}}(\boldsymbol{I} - \boldsymbol{C})^{-\mathbf{1}}$. This implies that the entries of $\boldsymbol{A}$ corresponding to the 8 countries mentioned above are

	ARG	BOL	BRA	CHL	DZA	NGA	PRY	URY
ARG	0.6688	0.0222	0.0164	0.0111	0.0062	0.0011	0.0308	0.0269
BOL	0.007	0.6673	0.004	0.0007	0.0001	0.0001	0.0016	0.0008
BRA	0.0596	0.0519	0.6706	0.0198	0.0069	0.006	0.0713	0.0594
CHL	0.0051	0.0423	0.0058	0.6678	0.0006	0.001	0.0162	0.0042
DZA	0.0004	0.0003	0.0025	0.0003	0.6673	0.0003	0.0004	0.0005
NGA	0.0026	0.0007	0.0053	0.0004	0.0005	0.6674	0.0008	0.0065
PRY	0.003	0.0028	0.0042	0.0027	0.0003	0.0002	0.6673	0.0036
URY	0.0019	0.0014	0.0019	0.0006	0.0004	0.0002	0.0035	0.6669

The next parameter that is required to be estimated using real data is the vector of primitive asset prices $\boldsymbol{p}$. This thesis follows the assumption that the income stream of each country is proportional to its gross domestic product (GDP) and it is a simple estimate for the fundamental value of the country's economy. Therefore, entries of the vector $\boldsymbol{p}$ are approximated by nominal values of GDP, normalizing the GDP of one of the countries to 1.

Initial ratings are straightforward to estimate as all major credit rating agencies classify most sovereign states around the world. This thesis uses S&P's long-term ratings data which can be collected from the S&P webpage and various publications. To keep the analysis uncluttered, the rating scale is converted according to Table 2. This partition of the ratings is a standard approach to look at "whole letter" ratings and it implies that a 10-state model is used to predict the effect of a shock. Each country's rating prior to the shock determines the corresponding number of threshold values and downgrade costs. The data also allows the comparison of the model prediction with actual, post shock ratings of the countries.

<table>
<tr><th>S&P</th><th>Model</th><th>S&P</th><th>Model</th></tr>
<tr><td>AAA</td><td>10</td><td>B+</td><td rowspan="3">5</td></tr>
<tr><td>AA+</td><td rowspan="3">9</td><td>B</td></tr>
<tr><td>AA</td><td>B−</td></tr>
<tr><td>AA−</td><td>CCC+</td><td rowspan="3">4</td></tr>
<tr><td>A+</td><td rowspan="3">8</td><td>CCC</td></tr>
<tr><td>A</td><td>CCC−</td></tr>
<tr><td>A−</td><td>CC</td><td>3</td></tr>
<tr><td>BBB+</td><td rowspan="3">7</td><td>C</td><td>2</td></tr>
<tr><td>BBB</td><td>RD</td><td rowspan="3">1</td></tr>
<tr><td>BBB−</td><td>SD</td></tr>
<tr><td>BB+</td><td rowspan="3">6</td><td>D</td></tr>
<tr><td>BB</td><td></td><td></td></tr>
<tr><td>BB−</td><td></td><td></td></tr>
</table>

Table 2: S&P ratings converted to a 1-10 scale

The estimates of the threshold values and the failure costs are the most difficult in the model. It is nontrivial how real data could be used to assign values to these for each country. Therefore, similar explicit forms are assumed for these two parameters as in the simulation section.

4.3 Model prediction

In order to illustrate how the model may be used to predict a cascade of downgrades in a real-world network of countries, the first step is to pick a historical shock that resulted in a downgrade of a chosen sovereign. As a first exercise, let us model the network-based contagion effects of the downgrade of Brazil in September of 2015 from BBB- to BB+. This corresponds to a drop from investment grade to junk grade and is equivalent to a rating change from 7 to 6 in terms of the 10-state model. During this period, Brazil also experienced an economic decline, a nearly 27% decrease in GDP between 2014 and 2016. In terms of the model, this means that the values in the initial equilibrium of the model are given by

$$\boldsymbol{v}^{initial} = \boldsymbol{A} * \boldsymbol{p}^{initial}$$

, where $\boldsymbol{p}^{initial}$ is the vector of normalized GDP values at the end of 2014 for the countries of the network and $\boldsymbol{A}$ is estimated according to the method described in the previous subsection. The initial ratings are given by the S&P ratings of the countries as of 2014Q4, converted to the 1-10 scale. Then, the shock is modelled as a change in $\boldsymbol{p}$ from the initial values to 2016 values of GDP. Here, the modelled shock is not exclusive to the asset of a single country, but the GDP of each country changes individually. The model does not capture the dependence structure among these changes in any form, instead every downgrade that happens in the first step of the algorithm is considered a direct effect of the shock. The length of the chosen time period to model the shock is also important. A too wide time window might overstate the direct effect and hide actual contagion effects, while a too short window may not lead to a sufficiently large shock to start a cascade. Nevertheless, the current modelling choice provides a useful illustration. As in the case of the simulations of the random network, the threshold values are defined by the single parameter $0 < \theta < 1$, which gives the lowest threshold of each country as $\underline{v}_i^1 = \theta * v_i^{initial}$. The distance between the subsequent thresholds are constant, such that for organization i with an initial rating of $q \geq 3$

$$\underline{v}_i^l = \underline{v}_i^{l-1} + \frac{1-\theta}{q-1} * v_i^{initial}$$

,where $l = 2, \dots, q-1$. In the 10-state model, the distance has to be chosen larger than in the 3-state model to have meaningful differentiation between the ratings, thus model prediction is performed for $\theta = 0.1, 0.2, \dots, 0.5$. For higher values of θ, more and more downgrades occur and eventually, all countries are predicted to end up in default, which is clearly unrealistic. The failure costs for i are assumed to be the same as in the 3-state model:

$$\beta_i^1 = \frac{1}{2} * \underline{v}_i^1$$

and

$$\beta_i^l = \frac{1}{2} * (\underline{v}_i^2 - \underline{v}_i^1)$$

, where $l = 2, \dots, q-1$. This concludes the calibration of all of the model parameters in the real-world scenario and thus all the ingredients are present to make the predictions using the same algorithm as previously.

Since the availability of the export, rating and GDP data are not uniform across the countries of the world, the trade network narrows down to 112 sovereigns. The previously mentioned countries that are most dependent on Brazil (BRA) and have data available are Argentina

(ARG), Bolivia (BOL), Chile (CHL), Nigeria (NGA), Paraguay (PRY) and Uruguay (URY). Table 3 presents the initial ratings and the predicted changes for different values of θ. It also shows the actual ratings after the shock, which are taken to be the 2016Q3 S&P ratings of the countries. The date for the post-shock ratings is another discretionary factor as it is nontrivial how quick contagion is in a real network. The direct effect of the shock is given by the downgrades from the initial ratings in the first step of the algorithm. These are due to the decrease in asset prices not because of the modelled network effects. The predicted rating changes come from the comparison of the rating of each country at the beginning of the second step of the algorithm to its final predicted rating. These can potentially show any true contagion effects captured in the model. For $\theta \leq 0.3$, Brazil's predicted direct downgrade from 7 to 6 is in line with the observed downgrade in 2015Q3 that the example is based on. When the value of θ is less than or equal to 0.2, the model predicts that there are no further rating changes for Brazil. However, when $\theta = 0.3$, the model predicts additional downgrades in the network, which are in fact a result of contagion: due to the direct downgrade of Brazil, Paraguay is downgraded from a rating of 6 to 5 and also, Brazil suffers a further downgrade to a rating of 5. This is an illustration of a cascade in a real network. For $0.4 \leq \theta$, the model predicts an excessively negative direct effect that results in a drop of two ratings for Brazil, which contradicts the actual rating change observed. As for the other countries, Argentina was in selective default (SD) between 2014Q3 and 2016Q2, when it was upgraded to B- after ending bond default. However, the prediction will always be that it stays in default as upward rating changes are not considered by the model. The ratings of Bolivia, Chile and Uruguay are not affected in any way according to the model, while Nigeria suffers a direct drop of at least one rating for all values of θ and an additional downgrade due to contagion for $\theta = 0.4$. The last column of the table allows for comparison of the predictions of the model and the actual ratings after the shock. In this example, the best fit is for the cases of $\theta = 0.1$ and $\theta = 0.2$, in which a cascade of downgrades does not emerge. The two changes between the initial and the actual ratings are fully explained by the direct effect of the shock in the model, thus indicating the absence of network-based contagion effects among these countries in this particular instance.

	initial rating	predicted rating changes					actual rating
		θ=0.1	θ=0.2	θ=0.3	θ=0.4	θ=0.5	
ARG	1	1→1	1→1	1→1	1→1	1→1	5
BOL	6	6→6	6→6	6→6	6→6	6→6	6
BRA	7	6→6	6→6	**6→5**	5→5	5→5	6
CHL	9	9→9	9→9	9→9	9→9	9→9	9
NGA	6	5→5	5→5	5→5	**5→4**	4→4	5
PRY	6	6→6	6→6	**6→5**	5→5	5→5	6
URY	7	7→7	7→7	7→7	7→7	7→7	7

Table 3: Model prediction: initial ratings, predicted changes and actual ratings for different threshold values (downgrades due to contagion are highlighted)

This exercise of the implementation of the model for a real network of interdependent organizations only serves as an illustration of the methodology. The previously mentioned issues of selecting the dates of observations for GDP values and actual ratings and also the rating scale conversion provide a limitation of the real-world application presented above. For example, Nigeria actually suffered two separate downgrades during this period from BB- to B+ and then to B. The latter is masked by the "whole letter" conversion of the credit ratings used in the model, and might be a network-based effect in reality that is captured as a part of an overstated direct effect in the model. More sophisticated estimates of the parameters would be required to achieve robust results and a profound conclusion. Nevertheless, the exercise demonstrates a simple application of the model and how it can be used to investigate the presence of contagion effects in credit rating migration. It also shows how the model might be used to better understand and isolate the networked portion of the effect of a hypothetical shock on credit rating changes. An example of this is presented in the following subsection.

4.4 Model forecast

The method presented previously can be also applied using a forward-looking approach. The parameters in the initial equilibrium can be calibrated using the most recent data available and the model can be used to forecast the credit rating changes of a set of countries in the new equilibrium that emerges as a result of a hypothetical shock.

At the moment of writing this thesis, the latest year for which the export values and credit ratings are available for a large set of countries is 2020. The corresponding trade matrix consists of 102 countries. For this exercise, initial asset prices are given by 2020 GDP values of the countries and the hypothetical shock is modelled as a drop in the price of the asset of a specific country. In particular, a shock is chosen that is sufficiently large to result in a downgrade of multiple ratings for Russia. The reason to focus on Russia is that at the time of writing the possible sovereign default of Russia is the biggest story in the sovereign debt space. The algorithm is used to model the cascade potentially emerging from a deep cut to Russia's sovereign rating and to predict how the ratings across the network would change in this scenario. Using the "whole letter" scale, the initial rating of Russia is 7 equivalent to BBB. First, assume a shock that leads to a direct downgrade of 3 ratings for Russia to CCC. For thresholds fixed at $\theta = 0.4$, the model predicts that the only country whose rating is affected by Russia's downgrade is Belarus which is downgraded from B (5) to CCC (4). Russia loses an additional rating in the cascade and ends up in CC (3) in the post-shock equilibrium. Next, consider a larger shock such that Russia defaults directly. The model predicts that besides Belarus, Latvia and Lithuania also suffer a "one letter" downgrade due to contagion. The list of the affected countries is not surprising in this case and is in line with one's expectations. In the second case, the model also predicts direct downgrades of Belarus and Lithuania in the first step of the algorithm, which actually highlights a limitation of the model. These downgrades are fully caused by the modelled economic shock and the strong dependence of these countries on the Russian economy, and not by downgrade contagion. However, in reality, the default of a country might not be the result of a recession, but it could also happen due to political instability, banking crisis or a country's inability to process payments in foreign currency. In the model the shock is always considered to be a decrease in asset prices $\boldsymbol{p}$, such other forms of the shock cannot be modelled in this framework. This again may lead to the exaggeration of the direct effect. Nonetheless, the above example shows that it is important to consider network-based effects when analysing credit rating changes or preparing stress-tests with downgrade scenarios.

5. CONCLUSION

This thesis has presented how a general model of a network of cross-holdings can be modified to study the behaviour of cascades of credit rating downgrades, which provides a more realistic framework than the vast majority of previous research on financial contagion. The main results build on an existing characterization of the network structure, but provide new insights about the mechanism of cascades. The first main finding is that there is a region of diversification of the network for which a cascade of single-rating downgrades can emerge, which is a feature of real networks as opposed to cascades of defaults. The second finding confirms the intuition that a network of organizations with more good countries can "fail bigger" due to downgrade contagion compared to a network of mostly bad countries. There still remains the study of the effect of thresholds and downgrade costs, which could be an interesting topic for further research. Also, the inclusion of the possibility of credit rating upgrades could be a further extension of the model that would allow the comparison of negative and positive contagion effects.

The thesis also gave an illustration of how the model might be applied to real networks to predict credit rating changes by incorporating network-based effects. The limitations of the methodology arising from the imperfect calibration and some improbable assumptions of the model were emphasized so that the actual predictions presented should not be considered flawless. Future research could focus on the remedy to these weaknesses and also study how the methodology of the thesis could be combined with other models of credit risk to incorporate network-based effects.

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7. GLOSSARY – SZÓSZEDET

cascade – kaszkád, lépcsőzetes sorozat

contagion – fertőzés

cost of capital – tőkeköltség

credit rating migration – hitelminősítés változás

creditworthiness - hitelképesség

cross-holding – keresztben tulajdonlás

default – csőd

downgrade – leminősítés

equilibrium – egyensúly

interdependence – egymástól függés

investment grade – befektetési minősítés

junk grade – "bóvli" minősítés

network – hálózat

network-based effect – hálózati hatás

out-degree – kifok

8. MAGYAR NYELVŰ ÖSSZEFOGLALÓ

A pénzügyi fertőzés természetének tanulmányozása nagy figyelemben részesült az elmúlt években, különösen amióta a rendszerszintű kockázatok és a pénzügyi rendszer együttes viselkedésének fontossága előtérbe került. Ez a szakdolgozat a hitelleminősítések potenciális fertőző hatásának jelenlétét és viselkedését vizsgálja pénzügyi hálózatokban. Az alapvető kérdés az, hogy lehetséges-e, hogy egy adott hálózatban lévő valamely vállalat vagy ország leminősítésének hatása tovább terjedjen a hálózat többi tagjára és leminősítések egy sorozata (kaszkád) alakuljon ki a vállalatok közötti kapcsolatok, és egymástól való pénzügyi összefonódások miatt. A szakdolgozat további célja, hogy megvizsgálja, hogyan függenek ezek a hatások a pénzügyi hálózat egyes tulajdonságaitól. Ezen mechanizmusok megértése elengedhetetlen, hiszen a mai globalizált világban a vállalatok hitelkockázatát nem lehet egyénileg, izoláltan vizsgálni, hanem figyelembe kell venni a pénzügyi hálózatok szereplői között fellépő hatásokat is.

A szakdolgozatban használt metodológia az Elliott et al (2014) által prezentált modell kiterjesztése. Az eredeti modell a vállalatok közötti pénzügyi kapcsolatok úgy írja le, mint egy hálózatot, ahol a vállalatok egymás részvényeinek valamekkora hányadát birtokolják. Tehát egy negatív sokk, ami csökkenti valamely vállalat értékét, szintén negatív hatással lehet más vállalatokra. Így modellezhető egy kezdeti csőd fertőző hatása a hálózat paramétereinek függvényében. Az eredeti modellben a vállalatok csak a csőd és a nem-csőd két állapotának egyikében tartózkodhatnak, és a szerzők csődök egy sorozatának mértékét vizsgálják. Azonban ilyen kaszkádok kialakulása meglehetősen ritka valós hálózatokban, ellenben hitelleminősítésekkel, melyek gyakran előfordulnak. Ezért a szakdolgozat legfontosabb újítása, hogy az itt használt modellben a vállalatok bármilyen hitelminősítéssel rendelkezhetnek, mint például csőd (D), C, BB, AAA, stb. Így lehetséges negatív sokkok következményeként kialakuló hitelleminősítések fertőző hatásának és a potenciális leminősítési sorozatok vizsgálata.

Elliott et al (2014) két paraméter mentén vizsgálja a hálózat struktúráját: hogy a hálózat mennyire összefüggő (diverzifikáció), illetve, hogy a vállalatok átlagosan milyen mértékben függenek a hálózat más tagjaitól (integráció). Ezeken kívül a szakdolgozatban használt modell alkalmas arra, hogy a hálózatban lévő vállalatok átlagos hitelképességének hatását is lehessen vizsgálni. A véletlen hálózatok szimulálásával kapott eredmények a következő új megállapításokat mutatják a 2-állapotú modellhez képest. Egyrészt, hogy van a diverzifikációnak egy szintje, ahol a hálózat már összefüggő eléggé ahhoz, hogy csődök egy kaszkádja ne tudjon kialakulni, de nem elég összefüggő, hogy egy leminősítési sorozat létre jöjjön. Ez az eredmény jobban illeszkedik a valóságban megfigyelhető hitelminősítés változásokhoz, ahol sorozatos csődök ritkák. Másrészt, amikor fertőzés végig tud terjedni egy hálózaton és leminősítések egy sorozata kialakulhat, akkor egy olyan hálózat, mely több magasabb hitelminősítéssel rendelkező vállalatból áll, sebezhetőbb, mint többségében rosszabb hitelképességű vállalatok egy hálózata. Az intuíció az eredmény mögött, hogy egy „jobb" hálózat nagyobbat tud bukni, ha fertőzés már kialakult.

Végül, a szakdolgozat illusztrálja, hogyan lehet a metodológiát alkalmazni valós hálózatok esetén. Az export értékek által konstruált nemzetközi kereskedelmi hálózat példáján bemutatja, hogyan lehet a modellt használni az egyes országok hitelminősítés változásainak becslésére, melyek egy gazdasági sokk hatására következhetnek be. A modell gyengeségei szintén meg vannak említve, amik óvatosságra intenek a konkrét előrejelzésekkel kapcsolatban.