## Elementary exercise solving (elemifmkrm22ga)

Goals: Developing a problem-solving routine, building on the core secondary school curriculum, occasionally filling in missing knowledge.
Knowledge: Knowledge of basic concepts, results and methods in the field.
Capability: Application of knowledge in the field, understanding of interrelationships and problem solving.
Attitude: Desire to improve mathematical knowledge and to learn as much as possible, and to apply knowledge as widely as possible.
Autonomy and responsibility: Formulate and analyse mathematical questions independently and evaluate the limits of their applicability responsibly.

Content:

Thinking methods (set theory, combinatorics, graphs) Algebra and number theory Functions, series Geometry Statistics, probability

## Elementary problem solving (elemipmkrm22ga)

Goals: Developing problem-solving routines, building on the core secondary school curriculum, and developing mathematical thinking.
Knowledge: Knowledge of basic concepts, results and methods in the field.
Capability: Application of knowledge in the field, understanding of interrelationships and problem solving.
Attitude: Desire to improve mathematical knowledge and to learn as much as possible, and to apply knowledge as widely as possible.
Autonomy and responsibility: Formulate and analyse mathematical questions independently and evaluate the limits of their applicability responsibly.

Content: Developing problem-solving thinking through primary and secondary school tasks. Systematising and applying elementary logical deductions and proof methods.

## Algebra and number theory (algsz1m22ea, algsz1m22ga)

(\*: only on intensive level)

Divisibility, primes, Fundamental Theorem of Arithmetic.
Number theoretic functions.
Congruences, theorems of Wilson, Euler and Fermat.
Reducing congruences to prime power, resp. prime modulus.
Elementary diophantine equations.
Primitive root, index, discrete logarithm.
Quadratic residues, Legendre symbol.
\* Elementary theorems on the distribution of primes, sum of reciprocals.

Complex numbers, geometric applications, roots of unity.
Polynomials, evaluation, roots, derivatives.
\* Cubic- and quadric equations.
Arithmetic of polynomials, testing rational roots, irreducibility criteria.
Cyclotomic polynomials, applications.
Interpolation.
Relations between roots and coefficients, symmetric polynomials.
\* Resultant, discriminant.
Basic notions of abstract algebra: group, ring, zero divisor.

Systems of linear equations, applications of Gaussian elimination.
Vectors, matrices, matrix operations.
Permutations, sign, cycle decomposition.
Determinant, volume with sign.
Laplace expansion, inverse matrix, Cramer rule.
Rank (row, column, determinant).

**Analysis 1 (analizis1m22ea, analizis1m22ga)**

sup, inf, real numbers, concept of limit, series (geometric, harmonic), continuity of functions, continuous functions on closed and bounded intervals, limit of functions, elementary functions, concept of derivatives, differentiation of elementary functions, differentiation rules, monotonicity, convexity, min-max problems, mean value theorems: Rolle's theorem, intermediate value theorem, L'Hospital's rule, Taylor polynomials, power series, Taylor series, e^x, solving elementary differential equations.

## Combinatorics 1 (kombin1m22ea, kombin1m22ga)

Proof methods, Counting, Binomial theorem, binomial coefficients, Pascal's triangle. Principle of inclusion - exclusion. Linear recurrences, Fibonacci numbers. Catalan numbers. BarKochba. Graphs, degree sequences, Trees, spanning trees, counting labelled trees. Eulerian walks, Hamilton paths and circles. Graph colourings. Planar graphs. Matchings, graph parameters, the König-Gallai theorems. Multiple connectivity. Ramsey theorems for graphs.

## Introduction to scientific computing (bevtudprogm22ea, bevtudprogm22ga)

Introduction to the Python programming language, basic programming concepts (variable, assignment, loops, conditionals, statements and expressions, functions), setting up an own programming environment, the interactive Python shell, packages, IDEs and editors.

Fundamental data structures, IO operations, mutable and immutable data structures, operations on strings

Programming paradigms, the fundamentals of the object-oriented paradigm, classes, methods, inheritance, operator overloading

Solution of basic algorithmic problems: least common divisor, binary search, recursion, dynamic programming and memorization, basic algorithms in graph theory

The scientific Python libraries: numpy, matplotlib, pandas. Linear algebraic problems, exploratory data analysis and visualization, analyzing of tabular data.

## Linear and abstract algebra (linalgm22ea, linalgm22ga)

Vector spaces
Subspaces
Dimension
Linear mapping, kernel, image
Linear transformation, determinant, invertibility
Rank
Diagonalizability, eigenvalues, eigenvectors
Characteristic and minimal polynomial
Invariant subspaces, Jordan form
Bilinear maps and forms. Quadratic forms.
Method of least squares.
Euclidean spaces and their special transformations.
Diagonalizability w.r.t. ONB
Extremal properties of eigenvalues.
Singular value decomposition.
Tensor product, symmetric and antisymmetric tensors.
Quaternions.

Groups of permutations, matrices, geometric transformations.
Subgroup generation. Cyclic group. Order of an element.
Lagrange Theorem and its corollaries. Cosets.
Enumerating symmetries via the Orbit-Stabilizer Theorem.
Burnside Lemma.
Isomorphism, homomorphism, kernel, image,
conjugation, invariant subgroup..

Algebraic and transcendental numbers, minimal polynomial.
Ideals, quotient rings.
E.g.. complex numbers, field with 9 elements.

## Analysis 2 (analizis2m22ea, analizis2m22ga)

Indefinite integral, techniques of integration, definite (Riemannian) integral, Fundamental Theorem of Calculus, problems leading to definite integral, improper integral, applications, series, tests, integral test: hyperharmonic series, sequences and series of functions, power series, pointwise and uniform convergence. Multivariable analysis, functions of several variables, graphs, curves, surfaces and their equations, neighbourhoods, limit and continuity of functions of several variables, Weierstraß' theorem. Multidimensional differential calculus, first order approximation, tangent plain, directional derivative, partial derivatives, gradient, level sets, vector fields, Jacobian, chain rule, multidimensional mean value theorem, Lagrange's inequality, implicit function theorem, inverse function theorem, higher order derivative, Hessian, Young's theorem, second order Taylor polynomials, local extremum, saddle point, conditional extremum, method of Lagrange multipliers

## Foundations of mathematics (matalapjaim22ga)

Naive set theory. Elements of axiomatic set theory. Axiom of Choice. Cardinals, operations with cardinals. Cantor's theorem and Russell's paradox. Cardinalities of some "well-known" sets. The axioms of real numbers. Ordered sets, well-ordering.
Normal level: Propositional calculus, truth functions, truth tables. Disjunctive normal forms, complete systems. Deduction, first order languages.
Intensive level: Introduction to first order logic.

## Geometry 1 (geom1m22ea, geom1m22ga)

Basic elements of the space. Length of a vector, Cauchy-Schwarz inequality, angle, orthonormal bases. Orientation of a vector space. Scalar, vectorial and mixed product, properties of these operations. Affine notions. Fundamentals of convexity, Helly's theorem. Euler's formula, platonic solids.

## Combinatorial optimization (komboptm22ga, komboptm22ea)

Fundamental graph optimization problems: shortest paths, minimum cost spanning tree, maximum matching in bipartite graphs. The traveling salesman problem. Approximation algorithms. Network flows. Linear and integer programming: applications.

## Differential equations 1 (diffegy1m22ea, diffegy1m22ga)

Separable equations, examples.
First order linear ordinary differential equations, examples.
Second order linear ordinary differential equations, harmonic oscillator.
Existence and uniqueness of solutions of first order ordinary differential equations.
Linear systems of ordinary differential equations, fundamental system of solutions.
Constant coefficient linear systems, matrix exponential.
Higer order ordinary linear differential equations.
Autonomous differential equations. Phase portrait, stability. Dynamical systems.
Notion of partial differential equation, examples.
First order partial differential equations, examples.
Heat equation, Cauchy problem.
Wave equation, Cauchy problem.
Poisson's equation, Laplace's equation. Boundary value problems. Maximum and minimum principles.
Initial value problems, Fourier series.

## Groups, rings and modules (csgymm22ea, csgymm22ga)

Cyclic groups and their subgroups.
Groups of small order.
Cayley Theorem.
Groups acting on sets
Quotient group, Homomorphism Theorem
Direct and semidirect product
Finite Abelian groups
Free group, Dyck's Theorem
Simple groups
Normal series, composition series, solvable groups
Centralizer, normalizer, center
Sylow's Theorems

Additive and multiplcative group of a ring.
Homomorphism Theorem for rings.
Quotient field, characteristic, prime field.
Orderable rings and fields
Number theory in domains.
Noetherian rings.
Hilbert Basis Theorem, application to invariant theory.
Rings with no left ideals.
Simplicity of full matrix rings over skew fields.

Modules.
Finitely generated modules over principal ideal domains.

Field extensions, multiplicativity of degrees.
Splitting fields exist. Uniqueness (without proof).
Finite fields.
Finite skew fields are commutative. (Wedderburn)
A finite ring with no zero divisors is a skew field.
Frobenius Theorem on real algebras.

General concept and existence of generated substructures.

## Fields and algebras (galoism22va)

Galois groups of field extensions, Fundamental Theorem of Galois Theory.
Galois groups of finite fields, quadratic reciprocity.
Geometric constructibility.
Solvability of equations with radicals.

Integral elements in ring extensions.
Hilbert's Nullstellensatz.
Transcendence degree.

Commutative diagrams, exact sequences.
Projective and injective modules.
Hom and tensor product.
Categories and functors.

Semisimple modules and rings.
Theorem of Wedderburn and Artin.
Group algebras, Maschke's theorem.
Quaternion algebras and quadratic forms.

## Number theory (szamelmm22va)

Gaussian integers, sums of two squares.
Sums of 3- and 4 squares, Waring's problem.
\* Binary quadratic forms and ideal classes of quadratic number fields.
\* Finiteness of the class number (positive definite forms), the class number 1 problem for imaginary quadratic fields (case of even discriminant).
Diophantine approximation.
Pell's equation.
Approximation of algebraic numbers, e and pi are transcendental.
\* p-adic numbers, Theorem of Hasse and Minkowski.
\* Elliptic curves, groups structure.
\* Hasse's estimate of the number of mod p points.

The Riemann zeta function and its relation to distribution of primes.
\* Special values of the zeta function.
Convolution and Dirichlet series of number theoretic functions.
The probability that two integers are coprime.
Dirichlet's theorem on primes in arithmetic progressions.

\* Sidon sequences.
\* Theorem of Cauchy and Davenport.
\* Covering systems.
\* Geometric number theory, the circle problem.
\* Hardy-Ramanujan Theorem.

Topics with \* may not come up in every year.

## Set theory and mathematical logic (halmlogm22ea, halmlogm22ga)

Transfinite induction, well-ordering theorem, application in various areas of mathematics.
A few fundamental results of logic: aximoatizability, Compactness Theorem, Gödel's theorems.

## Algebraic curves (algorbm22va)

The ring of polynomials, homogeneous polynomials, and of formal power series.

Affine and projective plane curves, germs of local singularities local intersection multiplicity.

Bézout theorem, applications.

Splitting lemmas, application (Pappus, Pascal)

Properties of curves of degree three, group structure

Inflection points, Hesse curve, the number of inflection points

The topology of smooth projective curves, genus formula

Local singularities, Milnor number, Milnor fiber

Topology of singular projective plane curves

Divisor class group, examples

The divisor class group of smooth curves of degree three

## Introduction to numerical methods (bevnumm22va)

Numerical differentiation and integration: midpoint and trapezoid rules
Arithmetic limitations, order of error
Solving linear equations: Jacobi iteration
Solving nonlinear equations: Newton's method
Piecewise linear and Lagrange interpolations
Eigenvalue problem: power method
Optimization: conjugate gradient method
Numerical solutions of differential equations: explicit Euler, implicit Euler with Newton's method

## Differential equations 2 (diffegy2m22ea, diffegy2m22ga)

Notions of stability, stability of linear systems.
Stability of nonlinear systems, linearization, Lyapunov's method.
Stability of periodic orbits, the Poincaré map.
Boundary value problems for second order linear ordinary differential equations.
Notion of distribution, operations on distribution, fundamental solution.
Classical inital value problems for the heat equation and for the wave equation.
Classical boundary value problems for elliptic partial differential equations, Green's function.
Sobolev spaces.
Weak solutions of elliptic boundary value problems.
Weak solutions of initial value problems.

## Numerical methods 1 (nummod1m22ea, nummod1m22ga)

Numerical differentiation
Arithmetic limitations, order of error
Solving linear systems: stationary iteration
Solving nonlinear systems: Newton type methods
Lagrange, piecewise polinomial and spline interpolations
Discrete Fourier transformation
Numerical integration: Newton-Cotes and Gaussian quadratures
Eigenvalue problem, singular value decomposition
Optimization: conjugate gradient method, least squares method
Numerical solutions of differential equations: explicit Euler, implicit Euler with Newton's method, basics of Runge-Kutta methods

## Functional analysis and its applications (funkalkm22ea)

Motivations of the the abstract treatment.
Function spaces as examples of Banach spaces.
Properties of Hilbert spaces, abstract Fourier series.
Bounded linear mappings: famous theorems and applications.
Famous types of operators. Properties of self-adjoint operators.
Solvability of operator equations in Hilbert space. Bilinear forms, Lax-Milgram-lemma.
Applications to integral and differential equations, weak solution of 1D boundary value problems.
Regular values, spectrum. The spectrum of compact self-adjoint operators, Hilbert-Schmidt series.

## Numerical methods 2 (nummod2m22va)

Numerical solutions of system of differential equations: Runge-Kutta methods, multistep methods, symplectic methods, adaptive methods
Elliptic boundary value problems: finite difference schemes
Multigrid methods
Numerical solutions of one dimensional advection and diffusion problems

## Operator and matrix algebras (omalg22ea)

Banach algebras and their elementary spectral theory.
Examples of Banach algebras: function, matrix and operator algebras.
Functional calculus: polynomial, entire function, and holomorphic functional calculus.
The Gelfand theory of commutative Banach algebras; an application: Wiener's theorem. Commutative C\*-algebras and continuous functional calculus.
Representations of Banach-\* and C-\* algebras, the noncommutative Gelfand-Naimark theorem.
Borel functional calculator for normal operators.

## Analysis 3 minor (analizis3m22ea, analizis3m22ga)

Curves, arclength, line integral, Newton-Leibniz theorem, the complex plane, complex power series, e^z, Euler's formula, z^2 and 1/z as transformations on the plane, square root function, complex logarithm function. Complex differentiability and its geometric meaning, complex line integral for piecewise C^1 curves, Cauchy's theorem for star domains (no proof), Cauchy's formula (no proof), residue theorem (no proof, only for finitely many points), some applications. Metric spaces, notion of topology, continuity, convergence, open-, closed sets, boundary point, connectivity, compactness, Weierstrass and Heine theorems, Banach's fixed point theorem. Multiple integrals, Fubini's theorem, Cavalieri's principle, applications. Integral transform, volume of a parallelepiped, the determinant as signed volume. Polar-, cylindric- and spherical coordinates, surface area, surface integral, existence and determination of potential functions, Green's theorem.

## Further topics in analysis (tovfejanam22ea, tovfejanam22ga)

· Normed spaces, C[a,b] space and series-spaces.
· Hilbert spaces, orthogonality, Riesz decomposition.
· Fourier series in Hilbert spaces.
· Continuous linear functionals in normed and Hilbert spaces
· Riesz representation theorem
· Eigenvalues and spectra
 · Fourier series, Fourier transform
 · Measurable spaces and mappings, measures
· Integration and neasure, function series. Interchanging limits and integrals.
· Extension of measures.
· Lebesgue and Lebesgue-Stieltjes measures.
· Absolute continuous and singular measures, Radon-Nikodym theorem.
· Product measures, Fubini's and Tonelli's theorem.
· L\_p spaces, completeness, convergence in L\_p and almost everywhere.

## Analysis 3 major (analizis3xm22ea, analizis3xm22ga)

Point sets, metric space, continuity, compactness.
Line integral, surface, surface integral, integral theorems.
Fourier series, Fourier transform.
Jordan measure, multiple integral.
Basics of functional analysis.

**Complex analysis (komplexm22ea, komplexm22ga)**

Complex differentiability, Cauchy-Riemann equations. Power series. Elementary
functions. The complex logarithm. Defining holomorphic logarithm.
Complex integration. Newton-Leibniz formula. The independece of the
Cauchy's fundamental theorem, Cauchy's integral formula. Sequences of
holomorphic functions. Expanding holomorphic functions into power series. The uniqueness theorem. The maximum priciple. Schwarz lemma. Bound for the coefficients of the
power series. Liouville' theorem. Fundamental theorem of algebra.
Laurent series, convergence domain, expanding holomorphic function
in Laurent series. Isolated singularities. Classification and description of isolated singularities. Casorati-Weierstrass theorem.
Residue theorem, and its applications. Evaluating improper integrals
and infinite sums. The argument principle, Rouché's theorem, local
value distribution of a holomorphic function. Linear fractions.
Conformal map from disks or half planes onto disks or half planes.
The Riemann mapping theorem about simply connected regions. Harmonic
functions. The Laplace equation. Connection with holomorphic
functions. Maximum and minimum principle. The Poisson formula.

**Measure theory (mertekm22ea, mertekm22ga)**

Lebesgue measure, Borel and measurable sets, abstract measure spaces, measurable functions and their integrals, limit theorems. Signed and complex measures, absolute continuous and singular measures, decomposition theorems, Radon-Nikodym theorem. Product measure, Fubini's theorem. Differentiation of measures, Lebesgue's density theorem. Lp function spaces, convolution.

## Functional analysis (funkanalm22ea, funkanalm22ga)

Normed, Banach and Hilbert spaces, continuous linear operators.
Function spaces.
Geometry of Hilbert spaces, abstract Fourier series.
Extension of linear functionals: Hahn-Banach theorems
Consequences of the completeness of Banach spaces: Banach-Steinhaus theorem, open mapping, closed graph and linear homeomorphism theorems
Spectral theory of continuous linear operators
Special linear operators on Hilbert spaces (normal, unitary, self-adjoint, and positive operators)
Compact operators: Hilbert-Schmidt theorem, Riesz's theory of compact operators, Fredholm alternative theorem.

## Fourier analysis (fourierm22va)

Fourier series: uniqueness, Dirichlet and Fejér kernels, Bessel's inequality and Parseval's identity. Fourier transform and inversion in the Schwartz-space, Plancherel formula. Discrete Fourier transform and FFT. Further selections from the modern theory and its applications.

## Introduction to operation research (bevopkutm22ea, bevopkut22ga)

Network optimization problems. Properties and solution methods of linear inequality systems. Linear/convex/integer optimization. Polyhedra, duality. Totally unimodular matrices and their applications.

## Operation research 1 (opkut1m22ea, opkut1m22ga)

Cheapest paths, matchings, flows, and circulations. Properties and solution methods of linear inequality systems. Linear optimization. Polyhedra, duality. Totally unimodular matrices and their applications. Introduction to convex optimization.

## Operation research 2 (opkut2m22ea, opkut2m22ga)

Simplex method, network optimization. Column generation, Lagrangean relaxation. Branch and bound. Applications of linear programming: approximation algorithms, game theory.

## Scheduling theory (utemezesm22ea)

Basic notions of scheduling theory. Single machine problems. Approximation algorithms, linear programming methods, combinatorial algorithms. Parallel, uniform, and independent machines. The shop model. Bin packing problems. Approximation schemes.

## Combinatorics 2 (kombin2m22ea, kombin2m22ga)

Generator functions, Stirling and Bell numbers.
Extremal graph theory.
Hypergraphs, set systems.
Block systems.
Linear algebraic methods.
Probabilistic method.
Polynomial methods.

## Optimalization in practice (optgybanm22ea, optgybanm22ga)

Selected applications of discrete and continuous optimization. Depending on the theme, topics may include continuous or combinatorial optimization techniques, and methods form economics. Application of network flow models, column generation, and branch and price methods. Practical examples: production planning, telecommunication, transport, manufacturing etc.

## Topics in geometry (fejgeomm22ea, fejgeomm22ga)

Euclidean space and Minkowski's space time. Foundations of spherical and hyperbolic geometry. The orthogonal and the Lorentz group, connections to the Special Theory of Relativity. Differential geometry of curves (possibly surfaces). Projective geometry. Quaternions and the SO(3) group. Convexity: polarity, n-dimensional polytopes. Hausdorff distance. Approximation of compact sets by polytopes and finite sets. Steiner symmetrization. Isoperimetric inequality. Geometric probability. Application: Johnson-Lindenstrauss flatenning lemma.

## Introduction to topology (bevtopm22ea, bevtopm22ga)

Metric spaces, metric continuity.
Notion of Topological space, examples, constructions.
Connectendness. Separation and Countability axioms.
Compact spaces. Manifolds, examples.
Euler characteristics.
Classification of compact connected surfaces.
Basic properties of the fundamental group, its calculation using covering maps.
 Applications of the fundamental group: Fundamental theorem of algebra, Brouwer fixpoint theorem, hairy ball theorem, subgroups of a free group.

## Geometry 2 (geom2m22ea, geom2m22ga)

n-dimensional Euclidean space and its isometries. The orthogonal group. Classfication of isometries in low dimensions. Group action, orbit. Poles of a finite subgroup of SO(3), order of a pole. Classification of finite subgroups of O(3). The symmetry groups of Platonic solids. Quaternions and SO(3). Classification of Platonic solids, Schläfli symbol in all dimensions. Volume and surface area of convex sets. Hausdorff distance. Completeness and compactness. Blaschke's selection theorem. Approximation of compact sets by polytopes and finite sets. Surface are of a convex polytope. Steiner's formula on the volume of the parallel body of a convex polytope. Steiner symmetrization, isoperimetric inequality.

## Differential geometry 2 (diffgeom22ea, diffgeom22ga)

Arc length of parametrized curves, natural parametrization. Frenet basis of a generic curve, curvature of curves, Frenet's formulas. The Fundamental Theorem of the Theory of Curves.

Parametrized surfaces. First and second fundamental form, Weingarten map. Normal curvature, principal curvatures and directions, Gauss-Kronecker curvature, Minkowski curvature. Equations of Gauss and Codazzi-Mainardi. Theorema Egregium. Bonnet's theorem. Geodetic curves. Gauss-Bonnet theorem.

## Algebraic topology (algtopm22ea, algtopm22ga)

Calculation of fundamental groups (Van Kampen theorem),
Introduction to the Theory of Knots,
 Homotopy groups, CW-complexes,
 Homology groups, Euler characteristics,
Differentiable manifolds, Lie groups,
Degree of maps,
Applications: Brouwer fixed point theorem, Poincaré-Hopf Theorem, Borsuk-Ulam Theorem.

## Projective and hyperbolic geometry (projhipm22ea, projhipm22ga)

Projective space obtained from a vector space, duality, homogeneous coordinates, cross ratio. Theorems of Desargues and Pappos. Combianatorial properties of finite projective spaces. The Fundamental Theorem of Projective Geometry, analytic description of polarities. Conic sections, quadratic surfaces, conjugate relation, pole, polar. Theorems of Pascal and Brianchon. Weak form of Bézout's theorem, linear system of spheres.
Axiomatic geometry, Legendre's theorems on angle sums. Parallelism, cycles, congruence, trigonometric formulas. Cayley-Klein model.

## Mathematical programming packages (matprogcsm22va)

SciPy, SageMath, NumPy, SymPy, Matplotlib, Pandas, Cython, GitHub, Sat solvers, LP and MIP solvers.

**C++ for mathematicians (cppmatm22va)**

Compiling, linking, running of programs. Preprocessor directives and tokens. Expressions and their evaluation. Declarations, functions, parameter passing. Class, operators, constructors, destructor. Copying objects. Templates. Inheritance, polymorphism, virtual functions.

## Computing methods in statistics (statszamm22ga)

Application of open-source softwares (R, possibly Python). Methods of data input, running the elements of the programme, and the interpretation of the results, with a focus on the following areas: descriptive statistics, hypothesis testing, goodness-of-fit, testing independence, analysis of variance, regression. Additional material: principal component analysis, canonic correlation, scaling, classification.

## Statistical models (statmodm22ea, statmodm22ga)

Statistical models as the system of assumptions on the model. Parametric vs non-parametris models. Significance. Regression models. Classification. Examples for non-independent samples: trend, period.

## Continuous modelling (folytmodm22va)

Introduction to the aim of modelling, to the methods for analysing continuous models, and to how to solve them by using numerical methods. Presentation of the mathematical tools for analysing and applying continuous models (in the deterministic as well as stochastic cases). Derivation and mathematical analysis of population models, epidemic models, immunological models, transport and flow models, financial models. Performing computer simulations based on the models.

## Data mining and machine learning (abgt22ea, abgt22ga)

- Introduction to data science and machine learning: exercises, examples, methodologies
- Probably Approximately Correct (PAC) learning, VC dimension, ERM, no-free-lunch theorem, Sauer's lemma, fundamental theorems of statistical learning
- Classification and regression, decision trees, k-NN, (naive) Bayes
- Logistic regression, regularization, LASSO, Tikhonov
- Dimension reduction, SVD, PCA, matrix factorization
- Support vector machines, kernel methods, RKHS, Gaussian regression
- Hybrid classification methods, bagging, boosting models, random forests
- Variable selection, filter-based, wrapper methods
- Cluster analysis (theoretical approaches, centroid-based, hierarchical, density-based, DBSCAN), model-based, Kleinberg's theorem
- Frequent pattern mining: apriori and its improvments, FP-growth

## Economical and financial mathematics (gazdpenzm22ea, gazdpenzm22ga)

Financial decisions, financial markets
Regular cash flows, annuities
Bond prices and risks (duration, konvexity, immunization)
Valuation of stocks
Risk and yield, portfolio theory (limit and effective portfolios) Markovitz, MAD as an LP problem
Yield computation, yield curve theories, risk measures
CAPM and tests, estimation of betas
APT, Fama French, multifactor models
Efficient markets, information, sigma algebras
Stock price models, Derivative products
Binomial tree, risk neutrality,martingale property and measure
Pricing vanilla call options in binary markets. Capital raising and decrease.
Arbitrage free and complete markets, uniqueness of prices
Trinomial tree, non-unique prices

## Deep learning and continuous optimization (mtfom22va)

1. Convex functions, local and global optima.
2. Convex program, Lagrange dual, KKT conditions.
3. Gradient method, Lipschitz gradient, exponential gradient, momentum method, adaptive version.
4. Newton's method, relation with gradient.
5. Local search, discrete convexity, simulated annealing.
6. Introduction, typical problems, applications.
7. Perceptron model, activation functions, multilayer neural networks, backpropagation.
8. Convolutional networks, image processing. Optimization in neural networks, stochastic gradient, momentum, AdaGrad, ADAM, loss functions.
9. Deep nets techniques: batchnorm, mini-batch, ResNet. Regularization: L1, L2, weight decay, early stopping. Dropout. Initialization strategies, transfer learning.
10. Generative modelling: GAN, VAE, linear factor models.
11. Natural Language Processing (NLP) basics. Vector models.
Sequence modelling and recurrent nets, LSTM.

## Analysis and complexity of algorithms (algbonym22ea, algbonym22ga)

Decision trees, sorting.
Finite automata.
Turing and RAM machines.
Dynamic programming.
Storage and search of graphs
(BFS and DFS).
Minimum cost spanning trees.
Shortest path.
NP-completeness and reductions.
Undecidability.

**Design of algorithms (algterv1m22ea, algterv1m22ga)**

Sorting and selection. Search trees and AVL trees. Hashing. Heaps and applications. Counting mod m. Dynamic programming. Data structures for graph. BFS and DFS. Minimum cost spanning trees. Shortest paths. Matching.

## Design of algorithms 2 (algterv2m22ea, algterv2m22ga)

Coding, entropy, data compression. DFS for directed graphs. Substring search. Randomized algorithms. Approximation algorithms. Fix parameter problems. Online algorithms. Parallel computing. Geometric algorithms.

## Computing (szamtud22ea, szamtud22ga)

Computational models;
Turing machine, RAM machine,
Computability theory, space and time;
P, NP and NP completeness,
 cryptography, Communication games

## Data science (adattudm22ea, adattudm22ga)

• High-dimensional data, similarity and distance measures, embeddings.
• Nearest neighbour problem, minhashing. locality-sensitive hashing, sketches, Johnson-Lindenstrauss theorem.
• Methods for data streams: sampling, Bloom filter, counting distinct elements, estimating moments.
• Classification and clustering with data streams.
• Large-scale machine learning: map-reduce model, linear algebraic and database tasks with map-reduce.
• Link analysis in networks.
• Recommendation systems, utility matrix, profiles, collaborative filtering, UV-decomposition.

**Introduction to probability (bevvalszm22ea, bevvalszm22ga)**

Probability field, random variables, expectation, variance. Absolutely continuous random variables, density functions. Random vector variables, covariance. Special discrete and absolutely continuous random variables. Convergence of random variables. Law of large numbers, central limit theorem. Random walks.

## Probability theory and statistics (valstatm22ea, valstatm22ga)

Convergence of random variables, properties. Strong law of large numbers. Borel--Cantelli lemma. Central limit theorem. Conditional expectation. Unbiased and consistent estimators, confidence intervals, method of moments, maximum likelihood estimate. Hypothesis testing,tests for normal sample. Linear regression.

## Probability theory 1 (valoszin1m22ea, valoszin1m22ga)

Probability field, random variables, expectation, variance. Absolutely continuous random variables, density function. Random vector variables, covariance. Special discrete and absolutely continuous random variables. Convergence of random variables. Law of large numbers, central limit theorem. Limit theorems. Random walks. Inequalities. Generating functions.

## Probability theory 2 (valoszin2m22ea, valoszin2m22ga)

Measure theory for probability. Kolmogorov's axioms. Random variables and vector variables. Convergence of random variables. Characteristic functions. Limit theorems. The general notion of conditional expectation. Martingales. Laws of large numbers. Series with independent terms.

## Mathematical statistics (matstatm22ea, matstatm22ga)

Statistical field, empirical distribution, Glivenko--Cantelli theorem. Sufficiency, completeness, unbiased estimates, information bound, effective and consistent estimates. Parameter estimation: moment method, maximum likelihood method, Bayesian mehtod. Hypothesis testing: tests, Neyman--Pearson lemma, testing normality, non-parametric methods. Confidence sets and confidence intervals. Linear and logistic regression, analysis of variance. Estimation of density functions.

## Introduction to stochastic processes (bevsztochfm22ea, bevsztochfm22ga)

Description of stochastics processes. Kolmogorov's theorem. Discrete-time Markov processes, with a focus on the case of finite state space (Perron--Frobenius theorems). Branching processes, probability of extiction. Ergodic theorem for strongly stationary processes. Renewal processes, elementary renewal theorem. Poisson process. Construction of Wiener process, properties of trajectories.

## Thesis tutorial (szakdolim22da)

This course provides the formal background for the work on the thesis and for the tutorials with the supervisor.