Megjelent

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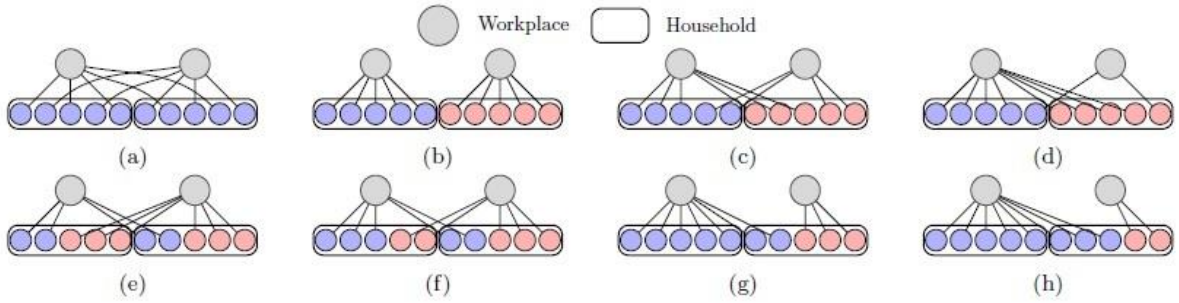
A terjedési folyamatokat gyakran hálózatokon modellezzük, ahol a jelek, információ, fertőzés két csúcs közötti kapcsolaton (élen) terjedhet. Azonban sok helyzetben ez nem elegendő a jelenségek leírásához. Például, gondolhatunk egy baráti társaságra, ahol egy fontos kérdésben a csapat egy tagján kívül mindenki egyetért. Egy hosszabb-rövidebb vita után (a csoportnyomás hatásának köszönhetően) lényegesen nagyobb annak valószínűsége, hogy a véleményével egyedül lévő tag meggondolja magát, esetleg kilép a csoportból, mint abban az esetben, ha párosával vitatnák meg ugyanezt a kérdést a társaság tagjai: úgy még esetleg terjedni is kezdhet a kisebbségben lévő vélemény. Ezt a jelenséget tehát csak páronkénti kölcsönhatásokkal nem tudjuk jól modellezni, ezért hipergráfokon folytatunk vizsgálatokat, ahol a hiperélek az emberek csoportjainak felelnek meg. Esetünkben kétféle csoport volt: háztartás, és munkahely, utóbbiakat véletlenszerűen alakítottuk ki. Azt vizsgáltuk, hogy a véleményváltoztatás, illetve munkahelyről való kilépés valószínűsége hogyan befolyásolja a folyamat alakulását, például a teljesen homogén csoportok (véleménybuborékok) kialakulásának sebességét, a stabil állapotok eléréséhez szükséges időt. Továbbá statisztikai elemzéseket is végeztünk, különböző paraméterbecslési eljárásokat összehasonlítva.

A kétrétegű, véletlen hipergráfon zajló adaptív modellünkben először egy kisméretű példát tanulmányoztunk, hiszen tíz csúcs esetén még az összes stabil állapot leírható. Ezek olyan konfigurációk, ahonnan indulva sem véleményváltoztatás, sem munkahelyről való kilépés nem történhet.

A nagyobb méretű modelleken, ezer csúcs esetén a változtatási valószínűségeket, mint ismeretlen paramétereket tekintettük. Négy különböző statisztikai módszert hasonlítottunk össze: lineáris regresszió, konvolúciós, illetve long short-term memory neurális hálózatok, XGBoost. Azt állapítottuk meg, hogy mind a négy módszer tud nagyon hatékony lenni megfelelő feltételek mellett, a modell különböző beállításaival, és az is előfordul, hogy a folyamat kezdeti, illetve stabilabb, polarizáltabb állapotában más-más módszer az optimális. Azt kaptuk, hogy az alapstatisztikához (az egyik vélemény részaránya) képest tovább információk is lényegesen hozzá tudnak járulni a pontossághoz.

Ábra:



Stabil állapotok a tíz elemű kétrétegű hálózatban

Adaptive models for spreading processes on two-layer random hypergraphs: from opinion bubbles to parameter estimation with machine learning methods

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When we study spreading processes, we often start with a network, on which signals, information, contagion can travel through pairwise connections. However, there are many real-world situations where more than two individuals interact and influence each other simultaneosly. To start with, we can think about a group of friends, where everyone except one person (Peter, say) has the same opinion about an important question. Then, after a discussion on this topic, Peter is likely to change his opinion, due to the well-known peer pressure phenomenon. On the other hand, if he meets his friends one by one, his view is more stable; he might even have some chance to persuade some of his friends to change their mind. These kinds of effects cannot be modeled by using pairwise interactions only. A similar situation appears in our brain: interactions of more than two neurons play a key role in biological processes (e.g. synchronization) that are necessary for a healthy functioning.

From the point of view of mathematics, group interactions can be modeled by hypergraphs. This is a mathematical structure consisting of a finite set of vertices V, and arbitrary subsets of V, called hyperedges. To model spreading processes, we can use differential equations, where some of the terms correspond to the hyperedges. From a probabilistic point of view, we can start with a random network configuration, and then individuals change their opinion based on the state of the hyperedges that they are in. The main idea is that individuals who represent a minority opinion in a hyperedge (corresponding to a group) have larger chance to change their opinion. In addition, we can introduce an adaptive (dynamical) version: an individual can quit the hyperedge and join another one, to avoid the situation of being in the minority. Such dynamics can lead to polarization: under appropriate circumstances, after a while, most groups will be homogeneous, corresponding to opinion bubbles.

In our work, in an adaptive opinion spread model, our first main goal was to study the effect of the parameters (which correspond to the stability of the opinion of an individual) on the global behavior of the system: on the speed of polarization, the probability that one of the possible opinions disappear, on the distribution of the workplace sizes. Then, we focused on statistical questions: how can we estimate the probability of opinion change or workplace quit, if we can observe only the frequency of each opinion, and do not have information about the individual events? For this, we quantitatively compared different machine learning methods, such as xgboost and convolutional neural networks.

Following our earlier studies about epidemic spread on two-layer random graphs, we considered a hypergraph with two types of hyperedges: household and workplace groups (although the latter do not strictly correspond to workplaces in real life). In our model, the household hyperedges are fixed: each individual lives in a family with five members, represented by a hyperedge of size 5. Then, uniformly at random, we distribute the individuals into workplaces of size 5, independently of the household structure. This will be the second layer of the hypergraph, but this will not be fixed:

individuals can leave their workplace and join a new one. To get the initial configuration, we assign opinion A or B independently to each vertex, with equal probability.

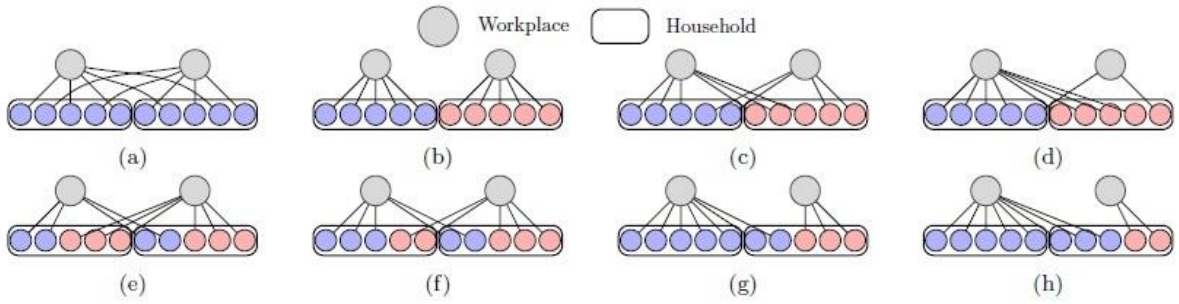
Our model has four parameters: β (opinion change parameter), q (workplace change parameter), r (threshold for the change parameters), λ (weight of workplaces). The dynamics is the following. At each step, we choose a vertex v uniformly at random, independently from the past. Let us assume that v's current opinion is A. Furthermore, let h and w denote the proportion of opinion A in the household (workplace, respectively) of v (including v itself).

Opinion change: We define $a=(h+\lambda w)/(1+\lambda)$. If a<r, then individual v changes opinion with probability $\beta(r-a)$.

Workplace change: If there was no opinion change in the first step, and $w<r$, then, with probability $q(r-w)$, vertex v quits its workplace. The choice of the new workplace is uniform at random among the workplaces where the opinion of v has a majority (that is, the proportion is strictly greater than 0.5). If there is no such workplace, the choice is uniform at random among all workplaces.

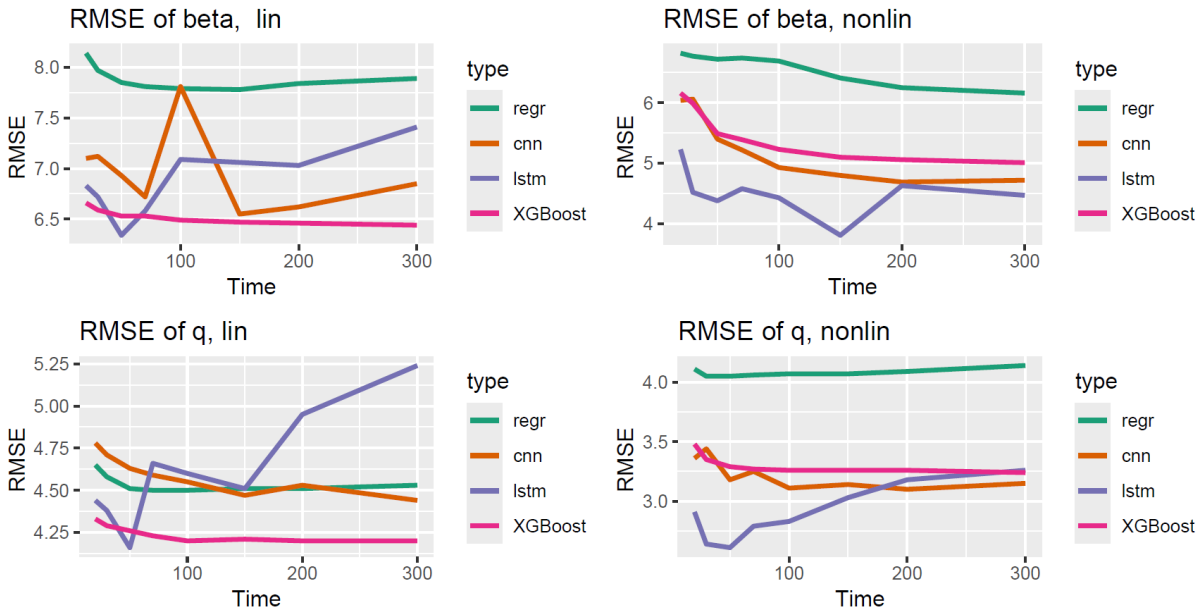
The first step corresponds to the situation of Peter described above. In many cases, we will use r=0.5, meaning that opinion change is possible only if the individual represents a minority opinion in its family. The second step guarantees the adaptivity of the model: not necessarily for workplaces, but for other communities, if Peter has a stable opinion and he is minority ($w<r=0.5$), he might consider leaving the group and find a new one. Notice that if β and q are larger, then groups where opinions A and B are present can disappear more easily, and we expect that the speed of polarization is larger. On the other hand, there is a strong connection between the change parameters: if β is large, then workplace change happens less frequently. It is also important that the smaller a and w are, the higher the probability of change; for example, peer pressure is stronger if Peter is alone with his opinion, compared to the case when he has at least one other member with the same opinion.

For a toy example with 10 vertices, we could describe all the absorbing states, and estimate their probabilities with computer simulations, as a function of β and q. In the figure below, we can see these states, with the joint structure of households and workplaces. As for the distribution, we found that the probability of the homogeneous states decreases if we increase q. With approximative calculations, we also showed that the time necessary for absorption is around a constant times $1/\beta$.



In the figure below, we can see a quantitative comparison of the estimation of β and q, by using the following statistical and machine learning methods: linear regression, convolutional neural network (CNN), long short-term memory method (LSTM), xgboost. Simulation for 1000 vertices were run, and

80% of these formed the training set, while 20% were added to the test set. The error is measured with the root mean squared error. We have two models: linear, when r=1, and nonlinear, when r=0.5 (as above). We can see that all methods can perform well under suitable condition, and it also happens that a different method is more effective in the first period of the process, compared to a second, more stable phase. We also examined the effect of hypergraph-based information (in addition to individual statistics) on the quality of the estimates.



Reference

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